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Macroeconomic Implications of Modeling the Internal Revenue Code In a Heterogeneous-Agent Framework*

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Abstract

Fiscal policy analysis in heterogeneous-agent models typically involves the use of smooth tax functions to approximate complex present tax law and proposed reforms. In this paper, we explore the extent to which the tax detail omitted under this conventional approach has macroeconomic implications relevant for policy analysis. To do this, we develop an alternative approach by embedding an internal tax calculator into a large-scale overlapping generations model that, while conditioning on idiosyncratic household characteristics, explicitly models key provisions in the Internal Revenue Code applied to labor income. We find that for a comparative-static steady state analysis of a given tax policy change, both approaches generate similar policy-induced patterns of macroeconomic activity despite variation in the underlying patterns of household tax-preferred consumption and labor supply behavior. However, this variation in underlying behavior is associated with significant quantitative and qualitative differences in macroeconomic aggregates along the transition path immediately following a policy change. Consequentially, although the use of unconditional smooth tax functions may be a reasonable modeling simplification for steady state analysis of tax policy, caution should be taken for their use in transition path analysis within heterogeneous-agent models.

JEL Codes: C63, E62, H30

Keywords: dynamic scoring; tax functions and calculators; heterogeneous agents

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1 Introduction

Fiscal policy analysis in macroeconomic models requires the incorporation of a tax system that can approximate complex present tax law and proposed deviations. One challenge is that policy changes affect each taxpayer differently. Early heterogeneous-agent models often approximate the tax system using a single parameterized tax function that smoothly maps household income into tax liabilities or effective tax rates, such as those developed in Easterly and Rebelo (1993), Gouveia and Struass (1994), Bénabou (2002), and Li and Sarte (2004).¹ While the recent work of DeBacker et al. (2018) builds upon this literature by using a large number of tax functions conditioning on household age, two shortcomings of this conventional approach remain: First, functional form assumptions and the imposition of smoothness on the tax system are questionable approximations of the actual tax system.² Second, failing to explicitly condition on idiosyncratic household characteristics such as filing status, number of dependents, and tax-preferred consumption choices ignores the variance in tax liability for households earning similar incomes.

To examine the extent to which the shortcomings of the conventional approach have implications relevant for policy analysis, we develop an alternative approach: Within a large-scale overlapping generations (OLG) model, we embed a tax calculator that explicitly incorporates tax provisions of the U.S. Internal Revenue Code (IRC) applied to labor income and conditions on idiosyncratic household characteristics when computing tax liabilities. Specifically, we model the present law³ statutory tax rate schedule, standard deduction, earned income credit, child tax credit, home mortgage interest deduction, state and local income, sales and property tax deductions, charitable giving deduction, net investment income and Medicare surtaxes, and dependent care credit. Unlike the conventional approach, we do not impose functional form assumptions on the effective tax rate schedules. Instead, endogenous household behavior generates deviations from the statutory tax rate schedule either by employment choices, which could affect tax credits, or consumption choices, which could affect deductions.⁴ This is important because policy proposals may include measures with potentially offsetting effects on incentives which vary across households (Gravelle and Marples, 2015). Our approach ensures that tax policy changes involving large, discrete effects on a relatively small group of households are not washed out as a smaller change for the wider population.

We choose one of the commonly-used smooth tax functions analyzed in Guner et al. (2014), the Bénabou (2002) tax function, as a benchmark. Relative to this, our approach offers two innovations: First, the internal tax calculator can target specific households affected by a policy change. Homeownership is a choice in the model, so if the home mortgage interest deduction were repealed, for example, only households claiming that deduction would experience an increase in their tax liability. Alternatively, the tax function would shift for every household, indiscriminately. The second advantage of the tax calculator is that households have the opportunity to *react optimally* to the policy change. If the deduction is repealed, renters who would purchase a house in the near future under present law may continue to rent, or decide to purchase a smaller property. Under the tax

¹Non-smooth tax functions have been used in Ventura (1999) and Altig and Carlstrom (1999).

²For example, Akhand (1996) provides evidence of misspecification for the Gouveia and Struass (1994) tax function.

³Our usage of ‘present law’ refers to 2018 tax law, which includes those changes to the IRC from passage of PL 115-97, colloquially known as the ‘Tax Cuts and Jobs Act of 2017’.

⁴For an early discussion on the use appropriate use of statutory and effective marginal tax rates, see Barro and Sahasakul (1983) and Easterly and Rebelo (1993).

function, the extent to which households are affected by the repeal is implicitly assumed.

We simulate two tax policy changes using the internal tax calculator and the smooth tax function: (i) a ten percent reduction in statutory tax rates on ordinary income; and (ii) an expansion of the earned income tax credit for childless adults. We find that for a comparative-static steady state analysis, the response of macroeconomic aggregates are similar across tax systems for both policy changes. However, variation exists in the underlying pattern of household tax-preferred consumption choices and labor supply behavior due to the explicit modeling of tax provisions with the internal tax calculator. This behavior is associated with significant quantitative and qualitative differences in macroeconomic aggregates along the transition path immediately following each policy change. This result indicates that while unconditional smooth tax functions may be an appropriate modeling simplification for performing steady state analysis with heterogeneous-agent models, they are less suitable for transition analysis of policy changes.

2 Model

In this section, we specify a large-scale OLG model —where market interactions take place between households, firms, a financial intermediary, and government—in which we embed a tax calculator to explicitly model IRC provisions that determine the tax treatment of labor income: Households make consumption, saving, labor supply, and residential choices. Firms hire labor and rent capital to produce a composite output good that can be transformed into either a consumption or residential good, or a financial asset. The financial intermediary takes in deposits of financial assets from households and allocates the funds to consumer and mortgage loans, federal government bonds, rental housing capital, and productive private business capital, remitting the return on this portfolio back to deposit-holding households. Federal, state and local governments collect tax liabilities owed by households and firms, and make consumption expenditures, public capital expenditures and facilitate transfer payments. While all agents have perfect foresight regarding the path economic aggregates, prices, and fiscal variables associated with a given policy, households face mortality risk.

The household sector is developed to exhibit heterogeneity along specific dimensions so that the tax calculator can explicitly model the desired IRC provisions applied to labor income. We include ex ante heterogeneity in age, labor productivity, family composition, and wealth endowments which imply ex post heterogeneity in residential choice and tenure, as well as the path of lifetime financial wealth. For the specification of several parameters and targets relevant for these dimensions, as well as for the internal tax calculator, calibration of the model relies heavily on the Joint Committee on Taxation’s Individual Tax Model (ITM), which makes use of data from individual tax returns filed with the Internal Revenue Service (IRS) and compiled by the IRS Statistics of Income Division.⁵ The calibration methodology is described in Section 3.

Exogenous population growth and technical progress are both present in the model. The framework is therefore structured to be consistent with the existence of a Balanced Growth Path (BGP), allowing for the application of stationary solution methods to compute equilibrium. The trend-stationary form of the model consistent with a BGP is specified as a dynamic program in Appendix B. Furthermore, since households face a set of decision-making frictions which generate decision rules that will be non-differentiable

⁵For a description of the Joint Committee on Taxation’s Individual Tax Model, see JCT (2015).

over some subsets of the state space, we rely on a discrete state-space solution method. The value function iteration - direct solution hybrid algorithm used to solve the model is described in Appendix C.

2.1 Demographics

The economy is populated by $J = |\mathbb{J}|$ overlapping generations of finitely-lived households where \mathbb{J} is the set of all possible discrete household ages with a maximum age of J . For working ages $j = \{1, \dots, R\} \in \mathbb{J}$, individuals in households decide how much to work, save, and consume on housing and non-housing goods and services. All individuals must retire by age R , but continue to make saving, housing and non-housing consumption choices for ages $j = \{R + 1, \dots, J\} \in \mathbb{J}$. Households are assumed to survive all possible working ages with a probability $\pi_j = 1$, and begin to face mortality risk upon reaching the maximum retirement age with the associated conditional probability of surviving from age j to $j + 1$ of $1 > \pi_j > 0$ until the maximum age of J , where $\pi_J = 0$ such that the household dies with certainty.

Each generational cohort of age j consists of discrete household groups, differing by age-varying labor productivity profiles, z_j^z indexed by $z \in \{1, \dots, nz\} \equiv \mathbb{Z}$, and family composition, $f = s$ for a single individual and $f = m$ for a married couple. In each period, $J \times nz \times 2$ distinct households make economic decisions.

The initial population is normalized to unity: $P_0 = 1$. The population grows exogenously at rate v_p . Letting $\Upsilon_P = (1 + v_p)$, the measure of total population at any time t is $P_t = \Upsilon_P P_{t-1} = \Upsilon_P^t$. The measure of household family composition f , age j and labor productivity type z is $\Omega_{t,j}^{e,f}$, so that at any time t , the population may be broken down by the measure of singles and married couples by age and productivity:

$$P_t = \int_{\mathbb{Z}} \int_{\mathbb{J}} (\Omega_{t,j}^{z,s} + \Omega_{t,j}^{z,m}) \, dj \, dz \quad (2.1)$$

While the age-productivity-family demographic distribution of households remains constant over time, the measure of each combination of attributes grows deterministically at the gross rate Υ_P .

2.2 Households

Households make consumption, labor, and residential decisions to maximize the present discounted value of their lifetime utility, which is derived from consuming non-housing and housing goods and services, and diminished from market work. The value function for a household of age j , with labor productivity type z , and family composition f , at time t , is $V_{t,j}^{z,f}(a_j, h_j^o)$, where beginning-of-period financial assets, a_j , and housing stock, h_j^o , are state variables.⁶ Each household optimally chooses future financial assets a_{j+1} , owner-occupied housing services h_{j+1}^o , and current market labor supply n_j , taking into account the intratemporal choices of ordinary consumption c_j^i , charitable giving c_j^g , and rental housing services h_j^r which maximize their instantaneous utility function $U_{t,j}^{z,f}$. Ordinary

⁶In Appendix B.2, we redefine the state space to permit numerical optimization over a single state variable, net worth $y_j \equiv h_j^o + a_j$, for each household of demographic (z, f, j) , and specify h_j^o and a_j as choice variables. Given that the problem is solved recursively by backwards induction as described in Appendix C, this change of variables does not alter the structure of the problem presented here.

consumption can be obtained from consumption of market-produced goods c_j^M or home production c_j^h .

Ordinary consumption and charitable giving are complimentary, and produce a non-housing consumption composite good c_j . Owner occupied housing and rental housing services are perfect substitutes, and summarized with a housing service composite good hs_j , as in Gervais (2002) and Cho and Francis (2011). As in the former work, we interpret housing services as the household's stock of durable goods plus the stock of residential capital. As in the latter work, households face a transaction cost ξ_j^H associated with changing residential status from a renter to a home-owner or vice versa to limit the frequency of status changes. The two composite goods are themselves nested into a third composite good x_j in a CES fashion. Leisure l_j is obtained from the portion of unitary time endowments not spent on market labor or non-market labor n_j^h .

To incentivize households to make charitable gifts, we assume a 'warm-glow' motive (Andreoni, 1989). Charitable gifts are made in terms of final goods, and are assumed to be received by agents outside of the model.⁷ The CES specification of the non-housing consumption composite, which includes both charitable gifts and ordinary consumption, is chosen because it allows for us to capture the empirically observed average level of charitable giving for each (f, z) demographic, although it understates the variance.

Market labor supply operates along both an extensive and intensive margin so that changes to aggregate employment can be broken down into movement of workers into or out of the labor force as well as changes in hours per worker. The former component is important because it has been determined as the primary driver of aggregate employment fluctuations (Kydland, 1995; Fiorito and Zanella, 2012). To avoid problems associated with the curse of dimensionality, we follow Chang et al. (2011) and specify indivisible market labor supply $n_j \in \mathbb{N} \equiv \{0, n^{PT}, n^{FT}\}$ such that individuals may choose between no work, part-time work, or full-time work. While somewhat restrictive, this specification allows us to capture the observation that working hours tend to bunch around part-time and full-time levels (Keane and Wasi, 2016).

Drawing from the observation that employed individuals spend less time on housework than the unemployed (Krueger and Mueller, 2012), we assume that the time each individual spends on home production exogenously varies inversely with their chosen quantity of market labor, so that the amount of home work is determined by the amount of market work through the function $n_j^h(n_j)$. Since housework can largely be outsourced, we further assume that all households derive the same value of home-produced consumption from each non-market labor hour through the function $c_j^h(n_j^h)$. Despite this simple structure of home production, its incorporation helps generate lifecycle heterogeneity in market labor hours across demographics as observed by Kuhn and Lozano (2008): First, higher earning individuals in the model tend to supply relatively more hours in the market than lower earning individuals. Second, as households age and become more productive in the market, variance of the net return to labor increases and drives greater variation in market hours at older ages.

Total costs to market work include a utility cost, a monetary cost, and a consumption cost. Following Holter et al. (2017), single households face a fixed utility loss of F^s if the individual enters the labor force, while married households face a fixed utility loss of F^m if the secondary earner works. The monetary cost $\kappa_j^{z,f}$ captures child-care costs

⁷Alternatively, it may be assumed that charitable gifts are made in terms financial assets which are then spent on final goods by the recipients. This would be equivalent to our specification if it is assumed that the recipients are non-taxable and operate costlessly subject to an intra-period balanced budget.

(Guner et al., 2011), and is a function the number of qualifying dependents within that household, $\nu_j^{z,f}$, and the market work hours of the single or secondary worker. This cost allows for the model to capture variation in lifecycle market labor and foregone earnings due to child-rearing (Adda et al., 2017). Finally, defining ordinary consumption as the sum of market and home-produced consumption goods, our specification of time use for home production implies that households face a loss in ordinary consumption which varies positively with market labor hours. Rogerson and Wallenius (2013) show that such a consumption cost helps to replicate the mostly abrupt nature of retirement observed empirically. The presence of this cost in our model can induce individuals of different demographics to choose full retirement at different ages prior to the required retirement age. Further, these features help generate the empirically observed extent and intensity of labor force participation across different demographic groups, without imposing exogenous lifecycle variation in labor disutility, see (Cogan, 1981).

Consider a single individual. The objective of this household's optimization problem for a known policy regime is:⁸

$$V_{t,j}^{z,s}(a_j, h_j^o) = \max_{\substack{a_{j+1}, h_{j+1}^o; \\ x_j, n_j \in \mathbb{N}}} U_{t,j}^{z,s}(x_j, n_j) + \beta \pi_j V_{t+1,j+1}^{z,s}(a_{j+1}, h_{j+1}^o) \quad (2.2)$$

$$U_{t,j}^{z,s}(x_j, n_j) \equiv \max_{h_j^r, c_j^i, c_j^g} \log(x_j) - \psi^s \frac{n_j^{1+\zeta^s}}{1+\zeta^s} - F^s \quad (2.3)$$

where:

$$y_j \equiv h_j^o + a_j \quad (2.4)$$

$$x_j \equiv (\sigma c_j^\eta + (1-\sigma) h s_j^\eta)^{1/\eta} \quad (2.5)$$

$$c_j \equiv (c_j^i)^{\theta^{z,s}} (c_j^g)^{(1-\theta^{z,s})} \quad (2.6)$$

$$h s_j \equiv \max\{h_j^o, h_j^r\} \quad (2.7)$$

$$n_j = \begin{cases} 1 - l_j - n_j^h(n_j) & \forall j \leq R \\ 0 & \forall j > R \end{cases} \quad (2.8)$$

$$F^s = \begin{cases} \phi^s & \text{if } n_j > 0 \\ 0 & \text{if } n_j = 0 \end{cases} \quad (2.9)$$

$$c_j^i \equiv c_j^M + c_j^h(n_j^h) \quad (2.10)$$

The functional form for instantaneous utility in equation (2.3) is chosen because it is consistent with a BGP in the presence of fixed utility costs from working.⁹

All households have their feasible choice set restricted by a budget constraint: The sum of expenditures on market consumption c_j^M , charitable giving c_j^g , and rental housing

⁸While prices, taxes and utility are time dependent, the household keeps track of choice variables over time using age. To reduce notational clutter, we omit the time subscript where able until aggregation.

⁹See Holter, Krueger, and Stepanchuk (2017) for the proof of balanced growth path consistency.

$p_t^r h_j^r$, as well as the choice of the next-period stock of financial assets a_{j+1} and owner-occupied housing h_{j+1}^o , can be no larger than the resources currently available to the household. Available resources consist of the gross return on beginning-of-period financial assets $(1 + r_t^p)a_j$ held by a financial intermediary, the after-tax bequests received from those who died at the end of the previous period beq_t , the stock of beginning-of-period owner-occupied housing less maintenance costs and economic depreciation $(1 - \delta^o)h_j^o$, the flow of income $i_{t,j}^{z,s}$ which is equal to labor income $n_j w_t z_j^z$ during working years and equal to social security payments $ss_j^{z,s}$ during retirement. These resources may be reduced by net tax liabilities $\mathcal{T}_{t,j}^{z,s}$, child-care costs $\kappa_j^{z,s}$, and housing transaction costs ξ_j^H . Formally the budget constraint takes the form:

$$c_j^M + c_j^g + p_t^r h_j^r + a_{j+1} + h_{j+1}^o \leq (1 + r_t^p)a_j + beq_t + (1 - \delta^o)h_j^o + i_{t,j}^{z,s} - \mathcal{T}_{t,j}^{z,s} - \kappa_j^{z,s} - \xi_j^H \quad (2.11)$$

where:

$$i_{t,j}^{z,s} \equiv n_j w_t z_j^{z,s} + ss_j^{z,s} \quad (2.12)$$

$$\kappa_j^{z,s} = cc^{z,s} \nu_j^{z,s} n_j \quad (2.13)$$

$$\xi_j^H = \begin{cases} \phi^o h_{j+1}^o & \text{if } h_j^o = 0 \\ \phi^r h_{j+1}^r & \text{if } h_j^o > 0 \end{cases} \quad (2.14)$$

and $cc^{z,s}$ is an exogenous scale parameter for cost per child. The determination of net tax liabilities $\mathcal{T}_{t,j}^{z,s}$ are detailed in Section 2.5.

Households are permitted to borrow and accumulate debt in excess of savings subject to the following restrictions:

$$y_j \geq \begin{cases} \underline{y}^{z,s} & \text{if } h_j^o = 0 \\ \gamma h_j^o & \text{if } h_j^o > 0 \end{cases} \quad (2.15)$$

where $\underline{y}^{z,s} < 0$ is the lower-bound of the net worth support and the parameter $0 \leq \gamma \leq 1$ can be interpreted as the down-payment ratio, or the minimum equity which a homeowner may hold in their home. This condition implies that while renters are able to accumulate unsecured debt up to the amount of $\underline{y}^{z,s}$, homeowners must maintain a minimum equity balance of γh_j^o . Homeowners have access to larger loans, using their home as collateral for borrowing up to $(1 - \gamma)h_j^o$.

Both rental housing and owner-occupied housing are subject to minimum sizes, where $\underline{h}^r < \underline{h}^o$ making rentals relatively more affordable. Similarly, ordinary consumption must be at least as large as a subsistence level \underline{c}^i . We further assume that households do not make charitable gifts if ordinary consumption is at this subsistence level:

$$hs_j \geq \underline{h}^r, \quad c_j^i \geq \underline{c}^i \quad (2.16)$$

$$h_j^o \geq \underline{h}^o \quad \text{if } h_j^o > 0 \quad (2.17)$$

$$c_j = c_j^i \quad \text{if } c_j^i = \underline{c}^i \quad (2.18)$$

As discussed in Section 2.5.2, the federal government will provide a conditional welfare transfer to any household that cannot afford both \underline{c}^i and \underline{h}^r .

It is assumed that households enter the economy with initial financial assets of a_1 and zero owner-occupied housing. Should a household live to the maximum age J , they are assumed to die with zero net-worth. This is not to say that savings, a_{J+1} , equal zero at the end of life, as owner-occupied housing capital could be positive, allowing agents to die with mortgage debt. In this way, the model allows for reverse mortgages. Formally, we impose the initial and terminal conditions:

$$y_1 = a_1 \quad (2.19)$$

$$h_1^o = y_{J+1} = 0 \quad (2.20)$$

$$V_{t,J+1}^{z,s} = 0 \quad (2.21)$$

Married households face a similar constrained optimization problem to that of single households. They make choices along the same margins to maximize the present discounted value of lifetime utility with the following objective function:

$$V_{t,j}^{z,m}(a_j, h_j^o) = \max_{\substack{a_{j+1}, h_{j+1}^o; \\ x_j, n_j^1, n_j^2 \in \mathbb{N}}} U_{t,j}^{z,m}(x_j, n_j^1, n_j^2) + \beta \pi_j V_{t+1,j+1}^{z,m}(a_{j+1}, h_{j+1}^o)$$

with the associated instantaneous utility function allowing for two potential earners:

$$U_{t,j}^{z,m}(x_j, n_j^1, n_j^2) \equiv \max_{\substack{a_j, h_j^r, h_j^o, \\ c_j^i, c_j^g}} \log(x_j) - \psi^{m,1} \frac{(n_j^1)^{1+\zeta^{m,1}}}{1 + \zeta^{m,1}} - \psi^{m,2} \frac{(n_j^2)^{1+\zeta^{m,2}}}{1 + \zeta^{m,2}} - F^m$$

Married households similarly face equations (2.4) through (2.21), indexed $f = m$ instead of $f = s$, with the exception of equations (2.8)-(2.10) and (2.12)-(2.13). First, equation (2.8) applies to labor supply for each of the married household's potential earners individually, so that n_j^1 and n_j^2 denote the primary and secondary labor supply where each worker gives up the same quantity of home production hours to work in the market. Second, since it is assumed that the costs associated with working apply only to the secondary earner, we have:

$$n_j^\epsilon = \begin{cases} 1 - l_j^\epsilon - n_j^h(n_j^\epsilon) & \forall j \leq R, \quad \epsilon = 1, 2 \\ 0 & \forall j > R \end{cases}$$

$$F^m = \begin{cases} \phi^m & \text{if } n_j^2 > 0 \\ 0 & \text{if } n_j^2 = 0 \end{cases}$$

$$\kappa_j^{z,m} = c c^{z,m} \nu_j^{z,m} n_j^2$$

Home production and income depend on the labor of both individuals in a household:

$$c_j^i \equiv c_j^M + c_j^{h,2}(n_j^{h,1}) + c_j^{h,1}(n_j^{h,2})$$

$$i_{t,j}^{z,m} \equiv (n_j^1 + \mu^z n_j^2) w_t z_j^{z,m} + s s_j^{z,m}$$

where $0 < \mu^z \leq 1$ is an exogenous productivity wedge between the primary and secondary workers. Note that the secondary earner's effective wage rate depends on the productivity-type specific term $z_j^{z,m}$, which also determines the primary worker's effective wage rate given the prevailing market real wage rate w_t . This specification is intended to capture both the observed earnings differential of workers within a married household and positive assortative mating (Greenwood et al., 2016, 2014; Eika et al., 2014).

Finally, since households can unexpectedly die when aged $R < j < J$, they may leave behind wealth that they have accumulated over their lifetime. Should a household die before reaching the maximum age J , a Λ proportion of their net worth is allocated to end-of-life consumption expenditures and the remaining $(1 - \Lambda)$ proportion is costlessly liquidated and collected by the government, taxed, and redistributed in a lump-sum fashion among the living. While this specification is chosen primarily so that the observed level of aggregate bequests can be targeted through the value of Λ , it also captures the observed increase in medical expenditures at the end of life (French et al., 2006). Furthermore, our perfect-foresight assumption implies that all agents can predict the quantity of wealth left behind by the dead, allowing for us to specify contemporaneous redistribution in the following manner:

$$beq_t = (1 - \Lambda) \int_{\mathbb{Z}} \int_{\mathbb{J}} (1 - \pi_j) \sum_{f=s,m} y_{t+1,j+1} \Omega_{t,j}^{z,f} dj dz - T_t^{beq} \quad (2.22)$$

where T_t^{beq} is the aggregate amount of taxes collected by the government on accidental bequests left at the end of period t .

2.3 Firms

Identical firms hire labor directly from households in a perfectly competitive labor market and rent capital from a financial intermediary to produce and sell output in a competitive goods market at profit maximizing levels. Production technology is assumed to be of the Cobb-Douglas form:

$$Y_t = G_t^g K_t^\alpha (A_t N_t)^{1-\alpha-g} \quad (2.23)$$

where $G_t = G_t^{fed} + G_t^{sl}$ the sum of beginning-of-period public capital owned by the federal government as well as the state and local governments, K_t and N_t are beginning-of-period productive private business capital and effective labor units used in production, and A_t is the level of labor-augmenting technological progress which evolves as $A_{t+1} = \Upsilon_A A_t$, where $\Upsilon_A = (1 + v_A)$ is the exogenous annual gross rate of technological growth. The final output good Y_t is the numéraire and can costlessly be transformed by households into a consumption good, owner-occupied housing services, or a financial asset as in Gervais (2002), Fernández-Villaverde and Krueger (2010), and Cho and Francis (2011), or into a charitable gift.

Aggregate effective labor N_t is the sum of efficiency-weighted labor hours:

$$N_t = \int_{\mathbb{Z}} \int_{\mathbb{J}} z_j^{z,s} n_{t,j}^{z,s} \Omega_{t,j}^{z,s} + z_j^{z,m} (n_{t,j}^{z,1} + \mu^z n_{t,j}^{z,2}) \Omega_{t,j}^{z,m} dj dz \quad (2.24)$$

where $\Omega_{t,j}^{z,s}$ is the measure of each productivity and age group for singles at time t , $\Omega_{t,j}^{z,m}$ is the corresponding measure for married households, and labor hours for singles, married primary and married secondary earners are $n_{t,j}^{z,s}$, $n_{t,j}^{z,1}$ and $n_{t,j}^{z,2}$ respectively.

Under this environment, the production sector can be modeled from the perspective of a representative firm which has the objective to choose the quantity of inputs K_t and N_t at factor prices r_t and w_t to produce output Y_t at a level that maximizes after-tax profits each period, taking the stock of public capital G_t as given. Assuming that wage expenses are fully deductible from all business-level taxes, profits Π_t for the representative firm are given by:

$$\Pi_t = \max_{K_t, N_t} \left\{ (1 - \tau_t^{bus})(1 - \tau_t^{slb}) (G_t^g K_t^\alpha (A_t N_t)^{1-\alpha-g} - w_t N_t) + \tau_t^{bus} lsd_t^{bus} - r_t K_t \right\} \quad (2.25)$$

where τ_t^{bus} and τ_t^{slb} are linear business-level tax rates applied to taxable profits at the federal level and the combined state-local level respectively, and lsd_t^{bus} is a lump-sum deduction against the federal tax liabilities. State and local business-level taxes are assumed to be fully deductible from federal business-level tax liabilities.

While the presence of public capital gives rise to economic rents, perfect competition in the goods market implies that firms will earn zero economic profits in equilibrium. To account for this we assume that the financial intermediary has sufficient market power over the firm to extract this rent, which accrues to the owners of capital. Firms then hire private factors at the given prices such that:

$$w_t = (1 - \alpha - g) G_t^g K_t^\alpha (A_t N_t)^{-\alpha-g} \quad (2.26)$$

$$r_t = \frac{1}{K_t} \left((1 - \tau_t^{bus})(1 - \tau_t^{slb})(\alpha + g)(G_t^g K_t^\alpha (A_t N_t)^{1-\alpha-g}) + \tau_t^{bus} lsd_t^{bus} \right) \quad (2.27)$$

2.4 Financial Intermediary

A representative financial intermediary pools the net stock of savings chosen by households into deposits to be invested in capital markets. Beginning-of-period aggregate deposits are expressed as:

$$D_t = \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} a_{t,j}^{z,f} \Omega_{t,j}^{z,f} dj dz \quad (2.28)$$

Each period the intermediary pays households the principle plus a portfolio return on their savings chosen in the previous period, $(1 + r_t^p)D_t$, and takes in new deposits D_{t+1} , for which it decides an investment allocation. The financial intermediary can invest in productive private business capital, K_{t+1} , rental housing H_t^r , and government bonds B_{t+1} . The equations of motion for these stocks are as follows:

$$K_{t+1} = (1 - \delta^K)K_t + I_t^K - \Xi_t \quad (2.29)$$

$$H_t^r = (1 - \delta^r)H_{t-1}^r + I_t^r \quad (2.30)$$

$$B_{t+1} = B_t + NB_t \quad (2.31)$$

There is a convex capital adjustment cost Ξ_t for deviating from the ‘break-even’ level of investment that would keep growth-adjusted private capital per capita constant:

$$\Xi_t = \frac{\xi^K}{2} \left(\frac{K_{t+1}}{K_t} - \Upsilon_P \Upsilon_A \right)^2 K_t \quad (2.32)$$

Net investment in business capital and rental housing chosen by the financial intermediary are I_t^K and I_t^r , while new bond issues by the government and purchased by the intermediary are denoted NB_t . While investment in business capital and government bonds at time t yields returns $(r_{t+1} + \delta^K)$ and ρ_{t+1} at time $t + 1$, investment in rental housing is assumed to be immediately available to households and thus yields contemporaneous return p_t^r . The intermediary incurs expenses from the economic depreciation of their stocks of business capital and rental housing capital, as well as potential business capital adjustment cost expenses. Table 1 summarizes the assets and liabilities held by the intermediary at the end of period t , as well as their contemporaneous income and expense flows over the period.

The objective of the financial intermediary is to choose the sequence of business capital and rental housing which maximize their net cash flow:

$$\begin{aligned} \mathcal{I}_t = \max_{K_{s+1}, H_s^r} \sum_{s=t}^{\infty} & (1 + r_s - \delta^K) K_s + (1 - \delta^r) H_{s-1}^r + p_s^r H_s^r + (1 + \rho_s) B_s + D_{s+1} \\ & - (1 + r_s^p) D_s - K_{s+1} - \Xi_s - H_s^r - B_{s+1} \end{aligned} \quad (2.33)$$

subject to the resource constraint:

$$D_{s+1} \geq K_{s+1} + H_s^r + B_{s+1} \quad \forall s \quad (2.34)$$

and the equations of motion for capital stocks, equations (2.29) through (2.31), with K_s , B_s , and H_{s-1}^r given.

The resource constraint (2.34) states that business capital and government bonds at the end of any given period plus current rental housing stock held by the intermediary must be less than or equal to the end-of-period stock of savings deposited by households. Given the objective of the financial intermediary, the price of rental housing is set so that the intermediary is indifferent between investing in rental housing or business capital, which yields the no-arbitrage condition:

$$p_t^r = r_{t+1} - \delta^K + \delta^r - \left(\frac{\partial \Xi_t}{\partial K_{t+1}} + \frac{\partial \Xi_{t+1}}{\partial K_{t+1}} \right) \quad (2.35)$$

We assume that government bonds, which are all held directly by the intermediary, enjoy a low ‘safe’ interest rate ρ_t relative to the return on productive capital. Since there is no uncertainty in this model, we accomplish this by introducing an exogenous, time-invariant wedge ϖ between ρ_t and r_t .

$$\rho_t \equiv r_t - \varpi \quad (2.36)$$

Finally, noting the contemporaneous income and expenses at time t from the income statement of the representative financial intermediary above, the imposition of a zero-profit condition on the financial intermediary implies that households receive a return on their deposits equal to:

$$r_t^p = \frac{(r_t - \delta^K)K_t - \Xi_t + p_t^r H_t^r - \delta^r H_{t-1}^r + \rho_t B_t}{D_t} \quad (2.37)$$

2.5 Government

2.5.1 Household Income Taxation

In this section we detail the tax treatment of household income, which involves the specification of a federal labor income tax, capital income tax, payroll tax, the special tax treatment of social security benefits as well as state and local taxes. We specify the general framework of labor income taxation under the internal tax calculator (ITC) developed in this paper and, for purposes of comparison, under the Bénabou Tax Function (BTF) (Bénabou, 2002). Since the goal of this paper to demonstrate the importance of the tax detail incorporated in the ITC for the tax treatment of labor income, we use the same framework for modeling all other taxes regardless of whether the ITC or the BTF is used in simulation.

To convert the household's economic income into a corresponding taxable income concept, a 'calibration ratio' is introduced to reflect the portion of a particular flow of economic income which may be subject to taxation. The productivity type - family composition specific calibration ratio for labor income, $\chi^{i,z,f}$, and ratio for capital income, χ^a , ensure the correct tax base. A household's adjusted gross labor income $\hat{i}_{t,j}^{z,f}$, and adjusted gross capital income $r_t^p \hat{a}_{t,j}^{z,f}$, are obtained by applying the corresponding calibration ratio, such that:

$$\begin{aligned} \hat{i}_{t,j}^{z,f} &\equiv \chi^{i,z,f} i_{t,j}^{z,f} \\ \hat{a}_{t,j}^{z,f} &\equiv \chi^a a_{t,j}^{z,f} \end{aligned} \quad (2.38)$$

Equation (2.39) summarizes the tax liability for a given household. Net tax liability $\mathcal{T}_{t,j}^{z,f}$ is equal to taxes owed on labor income, $tax_{t,j}^{z,f}$ — which may be determined either by the ITC or the BTF — plus tax liability on capital income, $\tau_t^a r_t^p \hat{a}_{t,j}^{z,f}$, plus tax liabilities associated with the Social Security system for retirees, $\tau_{t,j}^{pr} \hat{i}_{t,j}^{z,f}$, less federal transfer payments, $trs_{t,j}^{z,f}$, plus state and local tax liabilities, $slt_{t,j}^{z,f}$.

$$\mathcal{T}_{t,j}^{z,f} = tax_{t,j}^{z,f} + \tau_t^a r_t^p \hat{a}_{t,j}^{z,f} + \tau_{t,j}^{pr} \hat{i}_{t,j}^{z,f} - trs_{t,j}^{z,f} + slt_{t,j}^{z,f} \quad (2.39)$$

We now describe the specification of each fiscal instrument.

Taxation of Capital Income and Retirement Average tax rates on capital income are determined by age group - family composition specific tax functions: There is a unique function estimated each for working single, working married, retired single, and retired married household. We consider working-age households separately from retirees to more accurately capture the observed differential in capital income tax liabilities, since workers tend to save through tax-deferred savings vehicles. We assume that a household's average tax rate on capital income is a monotonically increasing function of their asset holdings

relative to the asset distribution $\mathbf{f}(a_t|f, j)$, which is conditional on family composition and whether the household is of working age or retired:¹⁰

$$\tau_t^a = \mathbf{q} \left(a_{t,j}^{z,f}, \mathbf{f}(a_t|f, j) \right) \quad (2.40)$$

We choose this specification over one that depends on a household's predetermined productivity type because asset income tends to have large variance over the lifecycle, which raises the potential for a household to move in and out of several different asset income quantiles over their lifetime.

Working households pay into the Social Security program at proportional payroll tax rate on labor income each period, which applies to all taxable labor income up to a specified threshold. Retired households pay a proportional tax on their receipts of Social Security income, which depends on the level of Social Security income itself. Formally:

$$\tau_{t,j}^{pr} = \begin{cases} \mathbf{p} \left(\hat{i}_{t,j}^{z,f} \right) & j \leq R \\ \mathbf{r} \left(ss_{t,j}^{z,f} \right) & j > R \end{cases} \quad (2.41)$$

Finally, we allow for taxes on accidental bequests left by deceased households. Since accidental bequests are redistributed in a lump-sum fashion among all living households, we specify that the tax rate τ_t^{beq} is linear in bequests and unrelated to either the benefactor or beneficiary household's other income.

Taxation of Labor Income: Internal Tax Calculator Under the ITC tax system, household tax liability on labor income, $tax_{t,j}^{z,f}$, is determined by application of a statutory marginal tax rate schedule, deductions, and credits. This mapping from choice variables, state variables and demographic characteristics to a tax liability is developed to be as close to the actual IRC as possible for the provisions modeled. The effective marginal tax rate on labor income is therefore not the statutory tax rate, but the marginal liability on incremental labor income after these deductions and credits have been applied.

The average tax rate on labor income before tax credits, τ_t^i , is determined by the statutory tax rate schedule in the tax calculator, adjusted gross labor income $\hat{i}_{t,j}^{z,f}$, adjusted gross ordinary capital income $krd_{t,j}^{z,f}$, and deductions $ded_{t,j}^{z,f}$. Deductions are a function of adjusted gross labor income, adjusted gross ordinary capital income, and tax-preferred consumption choices made by the household. Credits $crd_{t,j}^{z,f}$ are not only a function of labor income and family composition, but of other tax variables as well due to the refundability, or lack thereof, of various credits. Formally:

$$tax_{t,j}^{z,f} = \max \left\{ \tau_t^i \hat{i}_{t,j}^{z,f}, 0 \right\} - crd_{t,j}^{z,f} - tra_t^{z,f} \quad (2.42)$$

$$\tau_t^i = \boldsymbol{\tau}(\hat{i}_{t,j}^{z,f} + krd_{t,j}^{z,f} - ded_{t,j}^{z,f}) \quad (2.43)$$

$$krd_{t,j}^{z,f} = \mathbf{k}(r_t^p \hat{a}_{t,j}^{z,f}) \quad (2.44)$$

$$ded_{t,j}^{z,f} = \mathbf{d}(\hat{i}_{t,j}^{z,f}, krd_{t,j}^{z,f}, h_{t,j}^o, c_{t,j}^g) \quad (2.45)$$

¹⁰Since all households receive the same return on their savings, r_t^p , and face the same calibration ratio χ^a , an ordering of households by their financial asset holdings is equivalent to an ordering by the taxable income from these assets.

$$crd_{t,j}^{z,f} = \mathbf{c}(\hat{i}_{t,j}^{z,f}, krd_{t,j}^{z,f}, \kappa_{t,j}^{z,f}, ded_{t,j}^{z,f}) \quad (2.46)$$

$$tra_t^{z,f} \begin{cases} \neq 0 & \text{if } n_j > 0 \text{ and } f = s \\ \neq 0 & \text{if } n_j^1 > 0 \text{ and } f = m \\ = 0 & \text{otherwise} \end{cases} \quad (2.47)$$

where bold emphasis denotes a function. The last term in equation (2.42), is a productivity type - family composition specific transfer payment $tra_t^{z,f}$, which is used as a non-distortionary method of ensuring that households within a given (z, f) demographic group on average face a target average tax rate on labor income. This transfer may be positive or negative for different household groups, and is only nonzero for working households. Using equation (2.42) into equation (2.39) gives household tax liabilities when the treatment of labor taxation is as specified in the ITC tax system. Under this tax system, households consider the tax implications of their realized capital income when making their joint labor supply and savings decisions.

Taxation of Labor Income: Bénabou Tax Function This section describes the BTF tax system for labor income that we use as a benchmark for comparison to the ITC. The BTF is a commonly-used tax function (Guner et al., 2014; Heathcote et al., 2017; Holter et al., 2017) that generates smooth average tax rates and effective marginal tax rates over income. Like other commonly-used tax functions,¹¹ the BTF is continuously differentiable, allows for negative average tax rates to capture the effect of refundable tax credits, and is easily parameterized with the exogenous specification of an effective marginal tax rate and average tax rate at the desired level of aggregation.

Labor income tax liabilities under this tax system takes the form:

$$tax_{t,j}^{z,f} = \left(\hat{i}_{t,j}^{z,f} - \lambda_1^f (\hat{i}_{t,j}^{z,f})^{1-\lambda_2^f} \right) - tra_t^{z,f} \quad (2.48)$$

where the bracketed terms are the BTF, λ_1^f and λ_2^f are parameters which together determine the income-weighted average tax rate and effective marginal tax rate applied to labor income at for each family composition, and $tra_t^{z,f}$ transfers used as a non-distortionary method of ensuring that households within a given (z, f) demographic group on average face a target average tax rate on labor income subject to the conditions in equation (2.47). Using equation (2.48) into equation (2.39) gives household tax liabilities under the BTF tax system for labor income.

2.5.2 Federal Government

The federal government collects taxes from households and firms to finance non-valued government consumption expenditures, C_t^{fed} , investment in productive public capital, I_t^{fed} , social security payments for retirees, and other transfer payments to households. Budget deficits are financed through the sale of one-period bonds at the interest rate ρ_t . Total government expenditures must be less than or equal to total tax revenue net of transfer payments T_t^{fed} , plus new debt issues, less interest paid on old debt:

¹¹Other examples of smooth tax functions include, Easterly and Rebelo (1993), Gouveia and Struass (1994), Li and Sarte (2004), and DeBacker et al. (2018).

$$I_t^{fed} + C_t^{fed} \leq T_t^{fed} + B_{t+1} - (1 + \rho_t)B_t \quad (2.49)$$

where B_t is the beginning-of-period stock of outstanding one-period bonds. Total federal tax receipts T_t^{fed} are equal to the sum of current aggregate federal net tax receipts from households, T_t^{hh} , taxes collected on accidental bequests, T_t^{beq} , and the current federal tax receipts from firms, T_t^{bus} , so that $T_t^{fed} \equiv T_t^{hh} + T_t^{beq} + T_t^{bus}$. The law of motion for federal public capital is:

$$G_{t+1}^{fed} = (1 - \delta^g)G_t^{fed} + I_t^{fed} \quad (2.50)$$

The budget constraint in equation (2.49) implies: (i) in a macroeconomic steady state, the government rolls over a constant level of debt so that only finance charges ρB are paid each year, and (ii) during transition paths, government debt may grow or shrink. To rule out explosive debt paths, we maintain the no-Ponzi condition:

$$\lim_{k \rightarrow \infty} \frac{B_{t+k}}{\prod_{s=0}^{k-1} (1 + \rho_{t+s})} = 0 \quad (2.51)$$

which implies that the current stock of debt is equal to the present-discounted value of all future primary surpluses along any equilibrium path.

Retired households get an annual social security payment $ss_{t,j}^{z,f}$ from the federal government, which is assumed to be a function of OASDI-taxable average earnings over working years for their particular productivity type - family composition. The federal government also utilizes three other transfer instruments: First, households uniformly receive lump-sum transfers trl_t , which are intended to account for a specified share of current federal government transfers. Second, the government levies a lump-sum tax lst_t on households to incorporate those taxes specified to have only an income effect. Finally, to ensure that there exists a feasible choice set over the current net worth state space given the presence of lower bound constraints on housing and ordinary consumption, we specify conditional transfers $trw_{t,j}^{z,f}$ for the purposes of welfare and housing assistance to low-income households. Letting the right-hand side of the household budget constraint in equation (2.11) be denoted with $bdgt_{t,j}^{z,f}$:

$$trw_{t,j}^{z,f} = \max \{0, \underline{c}^i + p_t^r \underline{h}^r - bdgt_{t,j}^{z,s}\} \quad (2.52)$$

This means test requires a household both to be unable to afford the consumption and housing minimums, as well as have non-positive net worth in their next year of life in order to be a recipient of welfare transfers.

From a unified budget perspective, net income taxes collected by the federal government from households, T_t^{hh} consist of labor and capital income tax liabilities, payroll and retirement income tax liabilities, less transfers and social security payments:

$$T_t^{hh} = \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \left(tax_{t,j}^{z,f} + \tau_t^a r_t^p \hat{a}_{t,j}^{z,f} + \tau_t^{pr} \hat{i}_{t,j}^{z,f} - trs_{t,j}^{z,f} - ss_{t,j}^{z,f} \right) \Omega_{t,j}^{z,f} dj dz \quad (2.53)$$

where:

$$trs_{t,j}^{z,f} = trw_{t,j}^{z,f} + trl_t - lst_t$$

From equation (2.25), and the deductibility of state and local taxes, business-level federal tax liabilities T_t^{bus} take the form:

$$T_t^{bus} = \tau_t^{bus} ((1 - \tau_t^{slb})(Y_t - w_t N_t) - l s d_t^{bus}) \quad (2.54)$$

The federal government is assumed to collect all accidental bequests at the end of each period, and redistribute them in a lump-sum fashion the following period after applying a tax. Taxes collected on accidental bequests left at the end of the previous period are therefore:

$$T_t^{beq} = \tau_t^{beq} (1 - \Lambda) \int_{\mathbb{Z}} \int_{\mathbb{J}} (1 - \pi_j) \sum_{f=s,m} y_{t+1,j+1} \Omega_{t,j}^{z,f} dj dz \quad (2.55)$$

2.5.3 State and Local Government

Taxation at lower levels of government is incorporated both to account for their distortionary properties, for their deductibility from households' and firms' federal tax liabilities. For simplicity, we assume that household tax liabilities owed at the state and local level can be expressed as a proportion of taxable labor income and owner-occupied housing:

$$slt_{t,j}^{z,f} \equiv \tau_t^{sl\hat{z},f} + \tau_t^{slp} h_{t,j}^o \quad (2.56)$$

where τ_t^{sl} is a linear tax rate taken to represent potentially deductible state and local income and sales tax and τ_t^{slp} is a linear average tax rate on owner-occupied property. Aggregate household state and local tax liabilities can then be expressed as:

$$T_t^{slh} \equiv \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} slt_{t,j}^{z,f} \Omega_{t,j}^{z,f} dj dz \quad (2.57)$$

Aggregate business state and local tax receipts are:

$$T_t^{slb} = \tau_t^{slb} (Y_t - w_t N_t) \quad (2.58)$$

where τ_t^{slb} is the state and local business tax rate and the second term is business profits from equation (2.25). Total state and local taxes T_t^{sl} are equal to the sum of taxes collected from businesses and households: $T_t^{sl} \equiv T_t^{slb} + T_t^{slh}$. These receipts are assumed to be spent on non-valued state and local composite government consumption expenditures C_t^{sl} and investment in productive public capital I_t^{sl} subject to an intraperiod balanced-budget condition:

$$I_t^{sl} + C_t^{sl} = T_t^{sl} \quad (2.59)$$

where the law of motion for state and local public capital is:

$$G_{t+1}^{sl} = (1 - \delta^g) G_t^{sl} + I_t^{sl} \quad (2.60)$$

2.6 Equilibrium

In order to exhibit a balanced growth path in steady state equilibrium, the model must be transformed into trend stationary form. The transformation is described in Appendix B.1 and the model is respecified as a stationary recursive dynamic program in Appendix B.2. Equilibrium for a given tax system is formally defined in Appendix B.3 as a collection of household decision rules that maximize households' utility subject to household budget constraints, a collection of economic aggregates that are consistent with household behavior and the associated measure of households, profit-maximizing behavior by firms, a set of prices that facilitate clearing in factor, asset and goods markets, and an associated set of policy aggregates that are consistent with government budget constraints. In a steady state equilibrium, macroeconomic aggregates are growing at a constant rate equal to the sum of technological and population growth rates.

3 Baseline Calibration

We specify that discrete time in the model passes at an annual frequency. The set of parameters to be calibrated include both non-tax and tax policy parameters, both of which rely heavily on use of the Joint Committee on Taxation's Individual Tax Model (ITM) for specification, which makes use of data from individual tax returns filed with the Internal Revenue Service (IRS) and compiled by the IRS Statistics of Income (SOI) Division. We vary the use of long-run historical data, recent observations, and projections to construct parameter values in targeting the present economic environment and tax law as closely as possible for the initial steady-state baseline equilibrium, which we assume to correspond with calendar year 2018. We attempt to make the initial steady-state as comparable as possible across the internal tax calculator and the Bénabou tax function by imposing the same calibration targets on each, the key of which are summarized in Tables A1 and A2. To maintain focus on the tax details of our model, we discuss the calibration strategy for non-tax policy parameters in Appendix A; select exogenous parameters used are summarized in Table A3. The calibration of tax policy parameters is described in the following section.

3.1 Tax Policy Parameters and Targets

We let households of working ages $j = \{1, \dots, R\}$ coincide with actual ages 25 through 64. and retired households aged $j = \{R + 1, \dots, J\}$ coincide with actual ages 65 through 90. As discussed in Appendix A.1.3, we construct productivity types $z = \{1, \dots, 5\}$ to represent a notion of lifetime labor income quintiles for both single and married households respectively. We take the family composition $f = m$ to coincide with married households filing taxes jointly and, take $f = s$ to represent all single- and non-joint filing households. Finally, we consider the age of a given married household filing jointly to correspond with the age of the primary filer.

To map the flows of income received by households to their empirical counterparts, we consider: (i) gross labor income to correspond to the sum of a NIPA-comparable wage income concept described in Appendix A.1.3 and a $(1 - \alpha - g)$ share of pass-through business income, and (ii) gross capital income to correspond to the sum of interest income, dividends, realized capital gains, and an $(\alpha + g)$ share of pass-through

business income. We consider the business-level tax liabilities to correspond with those due by C-corporations.

The calibration ratios for labor and capital income, $\chi^{i,z,f}$ and χ^a , transform each household's economic income flow into a taxable base as specified in equation (2.38). We allow for productivity type - family composition specific calibration ratios for labor income to capture the variation in nontaxable labor compensation received by households over lifetime income quintiles. These are set exogenously as the ratio of taxable labor income to NIPA-comparable wage income as calculated by the ITM. There is a single calibration ratio for capital income, which allows for the model to generate the desired level of federal receipts from the capital income tax for a given specification of the capital income average tax rate function in equation (2.40). It is set endogenously so that aggregate capital income tax receipts relative to aggregate output in the model's present-law steady state equilibrium are equal to 2.04%, which is the level of total household capital income tax liabilities as calculated by the ITM relative to GDP in 2018 as projected by the Congressional Budget Office¹² (CBO).

3.1.1 Federal Government Taxation Household Income

Taxation of Capital Income and for Retirement Since all households face the same rate of return on deposits and calibration ratio applied to capital income, an ordering of households by capital assets is equivalent to an ordering of households by taxable capital income. We exploit this equivalence and specify an average tax rate function for capital income that depends on the relative location of a household's asset holdings in the cross-sectional distribution of assets, each for working single, working married, retired single, and retired married households as in equation (2.40). These functions allow households' tax liability on capital income, which may vary significantly over their lifecycle, to be independent of their permanent productivity type. We assume that this function takes a quadratic form:

$$\tau_t^a = q_{0j}^f + q_{1j}^f(a_{t,j}^{z,f}) + q_{2j}^f(a_{t,j}^{z,f})^2$$

These functions are fit to the observed present-law average tax rates ordered by taxable capital income as calculated by the ITM for each age group - family composition demographic. The functions are then mapped to the model-generated cross-sectional distribution of capital assets for each demographic, restricting tax rates to be non-negative. The average tax rate on capital income for a household is determined by evaluating this mapping at their relative level of capital asset holdings and demographic group (f, j) .

Taxes associated with the retirement system in our model include the OASDI portion of the payroll tax, which finances the Social Security system, as well as special tax treatment of the benefits received by retirees. The OASDI payroll tax applies to all labor income up to the threshold specified under present-law at a rate of 12.4%.¹³ The ratio for the taxable base of adjusted gross labor income adjusts endogenously and uniformly across taxpayers so that total payroll tax receipts relative to output are about 4.3%, as is projected by the CBO for 2018. To accurately capture the special tax treatment of Social Security income, the average tax rate on benefits in equation (2.41) for retirees is

¹² *The Budget and Economic Outlook: 2018 to 2028*, April 2018

¹³ While in practice employers and employees each remit payment of half the payroll tax liability, we assume the employee remits the full liability for simplicity.

set for each productivity type - family composition demographic to match the associated income-weighted average tax rates calculated by the ITM.

Due to uncertain lifespans, households will die leaving accidental bequests behind. To account for the high end-of-life costs including medical expenses, a Λ fraction of these assets will be allocated to consumption services. The remainder will be taxed at the average rate τ_t^{beq} and then distributed uniformly to the living population. The parameters Λ and τ_t^{beq} are jointly and endogenously determined so that average after-tax bequests received by households are approximately 0.9% of aggregate output¹⁴ while estate and gift tax revenue is approximately 0.12% of GDP, as projected by the CBO for 2018.

Taxation of Labor Income: Internal Tax Calculator Using the eligibility criteria stated in the IRC, we construct the internal tax calculator to directly model the following provisions: the statutory tax rate schedule for ordinary income, standard deduction, earned income credit, child tax credit, home mortgage interest deduction, state and local income, sales, and property tax deductions, charitable giving deduction, net investment income and Medicare surtaxes, and dependent care credit. In doing so, we explicitly account for any phase-in and/or -out regions of the tax provisions, which could cause substantial deviation of the effective marginal tax rate from the statutory marginal tax rate for some households. To construct each household's ordinary income tax base, we add a portion of taxable capital income to their taxable labor income; the portion of taxable capital income treated as ordinary is set to exogenously decrease over realized taxable capital income as computed from the ITM. Tax minimization behavior is assumed within the tax calculator such that any provision which may reduce tax liabilities is taken-up, including itemization of deductions (for state and local taxes, home mortgage interest, and charitable giving) in favor of the standard deduction.

The number of dependents claimed by a household matters for certain credits. While any qualifying dependent is counted for the earned income tax credit (EITC) calculations, the child tax credit applies to dependents under the age of 17, and the dependent care credit applies to dependents under the age of 13. Household averages for each of these three kinds of dependents are taken from the ITM and exogenously associated with each (j, z, f) demographic within the model. For some calculations the frequency distribution for the number of qualifying dependents within a particular household demographic is also used. The frequency distribution is especially important for modeling the EITC as it increases in the number of children nonlinearly.

By incorporating dependents in the model, we create another source of heterogeneity which has the potential to influence economic choices. For example, the data indicate that young, single households have fewer dependents on average than young, married households. If both young, single and married households would be in the EITC phase-in range at the same level of earnings, the marginal and average tax rates will be higher for the single household than for the married household as the EITC will be larger for the married household given more qualifying dependents. If the EITC amount is then changed for households with zero dependents, for example, then the single household will be affected more than the married household. The differential change to marginal and average tax rates could induce different behavior across these two households.

To impose discipline on the tax liabilities generated by the internal tax calculator, we use productivity type - family composition specific transfers $tra^{z,f}$, chosen so that

¹⁴This number reflects bequests that are left to recipients other than a spouse on average in the US from 1996-2012. See Wang (2016) for a summary of US bequest data.

income-weighted average tax rates on labor income generated by the model in either the present-law baseline or in the alternative-law scenario, $ATR^{z,f}$, match the income-weighted average tax rate targets generated by the ITM, $\overline{ATR}^{z,f}$. The income-weighted average tax rate on labor income generated by the internal tax calculator for a given working household of productivity type - family composition is:

$$ATR_t^{z,f} = \frac{\int_{\mathbb{J}} tax_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj}{\int_{\mathbb{J}} \hat{i}_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj} = \frac{\int_{\mathbb{J}} \left(\max \left\{ \tau_t^{i \hat{i}_{t,j}^{z,f}}, 0 \right\} - crd_{t,j}^{z,f} - tra_t^{z,f} \right) \hat{\Omega}_{t,j}^{z,f} dj}{\int_{\mathbb{J}} \hat{i}_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj}$$

where $\hat{\Omega}_j^{z,f}$ is a new measure constructed for this calculation which only weights households with positive labor income, therefore eliminating unemployed and retired households:

$$\hat{\Omega}_j^{z,f} = \begin{cases} \Omega_j^{z,s} & \text{if } n_j > 0 \text{ and } f = s \\ \Omega_j^{z,m} & \text{if } n_j^1 > 0 \text{ and } f = m \\ 0 & \text{otherwise} \end{cases} \quad (3.1)$$

Setting $ATR^{z,f} = \overline{ATR}^{z,f}$ solving the above equation for $tra^{z,f}$ gives us the desired level of productivity type - family composition specific transfer payments:¹⁵

$$tra^{z,f} = \frac{\int_{\mathbb{J}} \left(\max \left\{ \tau_t^{i \hat{i}_{t,j}^{z,f}}, 0 \right\} - crd_{t,j}^{z,f} - \overline{ATR}^{z,f} \hat{i}_{t,j}^{z,f} \right) \hat{\Omega}_j^{z,f} dj}{\int_{\mathbb{J}} \hat{i}_{t,j}^{z,f} \hat{\Omega}_j^{z,f} dj} \quad (3.2)$$

Taxation of Labor Income: Bénabou Tax Function The Bénabou tax function parameters $\{\lambda_1^f, \lambda_2^f\}$ are calibrated so that the income-weighted average and effective marginal tax rate for each family composition f , \overline{ATR}^f and \overline{MTR}^f respectively, match those generated in baseline equilibrium under the ITC. Given the specification of the BTF in equation (2.48), the income-weighted average tax rate for family composition f can be expressed as:

$$ATR^f = \frac{\int_{\mathbb{Z}} \int_{\mathbb{J}} tax_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj dz}{\int_{\mathbb{Z}} \int_{\mathbb{J}} \hat{i}_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj dz} = \frac{\int_{\mathbb{Z}} \int_{\mathbb{J}} \left(\hat{i}_j^{z,f} - \lambda_1^f (\hat{i}_j^{z,f})^{1-\lambda_2^f} \right) \hat{\Omega}_j^{z,f} dj dz}{\int_{\mathbb{Z}} \int_{\mathbb{J}} \hat{i}_j^{z,f} \hat{\Omega}_j^{z,f} dj dz}$$

where $\hat{\Omega}_j^{z,f}$ is defined as in equation (3.1). Setting $ATR^f = \overline{ATR}^f$ and rearranging for λ_1^f yields:

$$\lambda_1^f = (1 - \overline{ATR}^f) \left(\frac{\int_{\mathbb{Z}} \int_{\mathbb{J}} (\hat{i}_j^{z,f}) \hat{\Omega}_j^{z,f} dj dz}{\int_{\mathbb{Z}} \int_{\mathbb{J}} (\hat{i}_j^{z,f})^{1-\lambda_2^f} \hat{\Omega}_j^{z,f} dj dz} \right) \quad (3.3)$$

Analogously, the income-weighted effective marginal tax rate for family composition f can be expressed as:

$$MTR^f = \frac{\int_{\mathbb{Z}} \int_{\mathbb{J}} \left(\partial tax_{t,j}^{z,f} / \partial \hat{i}_{t,j}^{z,f} \right) \hat{i}_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj dz}{\int_{\mathbb{Z}} \int_{\mathbb{J}} \hat{i}_{t,j}^{z,f} \hat{\Omega}_{t,j}^{z,f} dj dz} = 1 - (1 - \lambda_2^f)(1 - ATR^f)$$

¹⁵Time subscripts are suppressed for steady state calculations.

Setting $MTR^f = \overline{MTR}^f$ and $ATR^f = \overline{ATR}^f$, the above equation can be solved for λ_2^f in terms of only exogenous tax rate targets:

$$\lambda_2^f = \frac{\overline{MTR}^f - \overline{ATR}^f}{\overline{ATR}^f - 1} \quad (3.4)$$

In the initial present-law baseline, we set $tra^{z,f} = 0$ under the BTF tax system.

A Brief Comparison of the ITC and the BTF: In Figure 1 we show the relationship between adjusted gross labor income and the portion of household tax liabilities attributed to labor income for both the ITC (top) and the BTF (bottom) in the initial present-law steady state equilibrium. The BTF smoothly maps taxable labor income into the associated tax liabilities, with the relationship determined by two parameters. The ITC, on the other hand, is a non-smooth mapping that allows for sources of variation in labor income tax liabilities other than taxable labor income. The IRC provisions explicitly modeled under this tax system allow variation in a household’s taxable ordinary capital income, number and age of dependents, and tax-preferred consumption choices all to affect tax liabilities attributed to labor.

Differences between the two tax systems are especially stark at the low and high ends of the income distribution. At the low end, tax liability falls below -\$5,000 for some households under the ITC, but never falls below -\$2,500 under the BTF. At the high end, married households earning the same amount of labor income, about \$355,000, have tax liability ranging from \$60,000 to \$71,000 under the ITC. Tax liability is approximately linear in adjusted gross labor income at this range under the BTF.

3.1.2 Federal Government Taxation of Business Income

We model a combined business sector without distinguishing between corporate and pass-through production for simplicity. Since only C-corporations have business-level tax liabilities, we assume that 73.44% of the combined business sector represents corporate entities owing tax, with the residual representing pass-through entities. This figure is the average corporate share of total value-added in production over 2007-2016 as computed from the Federal Reserve Bank Financial Accounts.

We take a linear tax rate τ_t^{bus} to represent the federal effective marginal corporate tax rate, and exogenously set it to 73.44% of the respective rate computed to the JCT’s Corporate Tax Model.¹⁶ We then set the federal lump-sum deduction lsd_t^{bus} endogenously so that federal business tax liabilities relative to aggregate output in the model’s present-law steady state equilibrium are equal to 1.19%, which is the projected federal corporate tax liabilities relative to GDP as projected by the CBO for 2018.

3.1.3 Federal Government Transfer Payments

A retiree’s current Social Security benefits depend on their past OASDI-covered labor income. Modeling this explicit dependence under the assumption of full rationality requires households to consider off-equilibrium paths with respect to social security benefits when labor supply decisions are actually made. Since this approach would add substantial computational time, we assume that households are boundedly-rational in this dimension

¹⁶For a description of the Joint Committee on Taxation’s Corporate Model, see JCT (2011)

and do not contemplate the effects on their future social security benefits when making current labor supply decisions. That is, labor supply choices —and hence past OASDI-covered labor income —are consistent with actual social current security benefits only for the on-equilibrium path for each (z, f) household demographic on average for a given cohort. Benefits are assumed to be a function of average lifetime earnings according to the benefit calculator available from the Social Security Administration.¹⁷

The purpose of welfare transfers $trw_{t,j}^{z,f}$ in our model is to ensure there exists a feasible solution at very low levels of consumable resources in the presence of positive lower bounds on housing and non-housing consumption. These transfers are subject to the means test in equation (2.52), which allows for a household without savings, and whose labor income is not high enough to afford consumption and rent minimums, to consume \underline{c} and \underline{h}^r . The lump-sum transfer trl_t is uniform across all households and set endogenously to about 3.07% of aggregate output, which reflects the CBO’s projection¹⁸ for the share of current federal government transfers unrelated to transfers for OASI and Medicare, as well as the outlay portion of refundable tax credits for 2018. Lastly, the lump-sum taxes lst_t are set endogenously to about 0.71% of aggregate output, which reflects the CBO’s 2018 projection for excise tax liabilities and miscellaneous penalties.

3.1.4 State and Local Government Taxation

We incorporate household state and local tax liabilities using the linear tax rate τ_t^{sl} applied to households’ adjusted gross labor income and the linear tax rate τ_t^{slp} applied the value of owner-occupied housing. The former rate is exogenously set to an effective rate representing the greater of state and local tax income or sales tax liabilities for each tax unit as computed by the ITM for 2018. The latter rate is exogenously set to $0.0105 \times 0.7174 = 0.0075$, which is the product of the national average property tax rate computed using state-level estimates from the National Association of Homebuilders for 2010-2014, and the the average portion of total residential capital that is not consumer durables as reported by NIPA for 2007-2016. The latter term in the product is included to account for the fact that our definition of housing services includes consumer durables.

For business-level taxation at the state and local level, we set the linear tax rate τ_t^{slb} endogenously so that state and local business tax liabilities relative to aggregate output in the model’s present-law steady state equilibrium are equal to 0.38%, which is the historical average state and local corporate tax liabilities relative to GDP as computed from the National Income and Product Accounts over 2007-2016.

4 Policy Experiments

We demonstrate the implications of explicitly modeling tax provisions applied to labor income by simulating two policy changes in turn: (i) a ten-percent reduction in statutory tax rates applied to ordinary income, and (ii) an expansion of the earned income tax credit (EITC) for childless adults. To this end, we assume in all of the simulations that any tax provision set to expire after 2018 are instead permanent, including the major

¹⁷While in practice OASDI covered earnings from the highest 35 years are used in the benefit calculation, for simplification purposes we assume benefits depend on the full 40 years of working life for households. See <https://www.ssa.gov/pubs/EN-05-10070.pdf> for a description of the benefit calculation.

¹⁸*An Update to the Budget and Economic Outlook: 2017-2027*

provisions associated with P.L. 115-97 which are set to expire in 2026.¹⁹

Steady State Analysis: We compare the economic and welfare differences from the initial present-law steady state to the alternative-law steady states across each tax system: Changes to key macroeconomic aggregates across steady states are reported as percent differences from initial steady state to alternative steady state. In measuring changes to welfare for households, we compute the proportional change to the lifetime path of the consumption composite x_j for households born in the alternative steady state that would be needed to make them indifferent between being born in either steady state. For a single household this is:

$$\sum_{j=1}^J (\beta^{j-1} \pi_j) U_j^{z,f} ((1 + \omega^{z,f}) x_j^{\mathbf{A}}, n_j^{\mathbf{A}}) = \sum_{j=1}^J (\beta^{j-1} \pi_j) U_j^{z,f} (x_j^{\mathbf{I}}, n_j^{\mathbf{I}}) \quad (4.1)$$

where $\omega^{z,f}$ is the proportional change in the consumption composite required to obtain indifference between steady states, and the \mathbf{A} and \mathbf{I} superscripts denote equilibrium choice variables associated with the alternative and initial steady states respectively.²⁰ A negative value of $\omega^{z,f}$ implies that a household demographic (z, f) would prefer birth in the final steady state relative to the initial steady state. To compute changes to aggregate welfare, we integrate both sides of equation (4.1) for both single and married households over productivity type and age:

$$\begin{aligned} & \int_{\mathbb{Z}} \sum_{j=1}^J \beta^{j-1} \pi_j \left\{ U_j^{z,s} ((1 + \bar{\omega}) x_j^{\mathbf{A}}, n_j^{\mathbf{A}}) \Omega_j^{z,s} + U_j^{z,m} ((1 + \bar{\omega}) x_j^{\mathbf{A}}, n_j^{1,\mathbf{A}}, n_j^{2,\mathbf{A}}) \Omega_j^{z,m} \right\} dz \\ &= \int_{\mathbb{Z}} \sum_{j=1}^J \beta^{j-1} \pi_j \left\{ U_j^{z,s} (x_j^{\mathbf{I}}, n_j^{\mathbf{I}}) \Omega_j^{z,s} + U_j^{z,m} (x_j^{\mathbf{I}}, n_j^{1,\mathbf{I}}, n_j^{2,\mathbf{I}}) \Omega_j^{z,m} \right\} dz \end{aligned} \quad (4.2)$$

where $\bar{\omega}$ is the aggregate proportional change in the consumption composite needed to make the households indifferent — on average — to being born in either steady state.

For purposes of our steady-state analysis, federal debt is held fixed while non-valued government consumption expenditures adjust to fully balance the federal budget. The choice of this instrument over lump-sum transfers avoids the income effect on household labor choice that would occur if lump-sum transfers were used to balance the budget. In holding federal debt fixed across steady states, we abstract away from the variation in the extent of crowding-out (crowding-in) that may occur under each tax system due to different paths of debt accumulation (deaccumulation).

Transition Analysis: We analyze the transition path immediately following the implementation of alternative tax policy beginning from an initial steady state associated with present tax law in 2018. To compare responses across tax systems we report changes to key macroeconomic aggregates over the first ten years following the policy change to

¹⁹See JCT (2018) for a list of expired and expiring tax provisions for years 2016 through 2027.

²⁰Since the consumption composite x_j nests housing and non-consumption consumption levels in a unitary homogeneous fashion, one can interpret $\omega^{z,f}$ as the required proportional change in all nested variables simultaneously.

coincide with the ‘budget window’ used by the United States Congress to inform legislative decision-making. For each tax system, the series are expressed relative to initial steady state levels.

Following the announcement and implementation of a policy change in ‘year 1’, which is assumed to be unanticipated in the ‘year 0’ initial steady state, agents in the model have perfect foresight regarding the future time path of policy and the economy. Any policy-generated federal budget deficits or surpluses are financed by borrowing or used to pay down existing debt for the first 30 years following a policy change. Non-valued government consumption expenditures and lump-sum transfers subsequently adjust by equal amounts to maintain long-run fiscal sustainability, which is phased in over years 31 through 40, so that the Federal budget is balanced thereafter. A sufficient number of time periods is chosen for the transition path so that the economy reaches a new steady state associated with alternative tax law.

4.1 Tax Instruments and Policy Changes

Both tax systems are calibrated for a policy change by holding constant income and choice variables associated with the initial steady state present-law equilibrium, and adjusting tax instruments to target the total conventional revenue effect over 2019-2024 as calculated by the ITM.²¹ Changes to labor income taxation under the ITC are explicitly incorporated in the tax calculator as specified by the policy change. Under the BTF, changes to average and effective marginal tax rates applied to labor income are made by parameterizing the tax function to match those changes predicted by the ITM for each f demographic. In both tax systems, transfers $tra^{z,f}$ are changed to target the distribution of the conventional revenue effect across (z, f) demographics as predicted by the ITM. For both tax systems, equation (3.1) is re-estimated to match the changes to average tax rates applied to capital income predicted by the ITM for the capital income quintile specific to the each (f, j) demographic, with the change scaled to match predicted change to capital income tax liabilities.

To evaluate how well the aggregate behavioral incentives associated with a given policy change are captured in each tax system, we compare the changes to aggregate average tax rates and effective marginal tax rates to the analogous changes predicted by the ITM. In Table 2, we report errors for the *changes* to income-weighted average effective marginal tax rates applied to labor income, holding constant present-law equilibrium household income levels and choice variables. The small errors indicate that both tax systems comparably replicate the tax rate changes predicted by the ITM. Consequentially, we have confidence that the tax instruments available in both tax systems are calibrated well to capture the aggregate behavioral incentives associated with each policy change.

4.2 Policy Experiment 1: 10% Statutory Tax Rate Reduction

The first policy experiment is a permanent ten percent reduction in statutory tax rates for ordinary income – which include wage income, interest income, short-term capital gains, nonqualified dividends, and pass-through business income. While a small portion of the conventional revenue effect from this policy is due to the change in tax treatment of capital income, nearly all of it results from the change in tax treatment of labor income.

²¹The conventional revenue effect is the estimated change in tax receipts from those projected under a present law baseline forecast, holding constant gross national product. See JCT (2011) for more details.

4.2.1 Steady State Comparison

A comparison of select aggregates across steady states is shown in the two left columns of Table 3 for both tax systems. In each case, effective (productivity-weighted) labor supply increases, and firms respond to the increased marginal product of capital by increasing their use of it in the expanded production of output, all in similar magnitudes. Since budget short-falls are completely financed by a reduction in non-valued government consumption in the alternative steady state, private capital is not crowded-out by public debt accumulation. An increased after-tax wage rate results, leaving households with sufficient resources to increase their consumption of market goods and housing services.

While use of the ITC or BTF is unimportant for the particular changes to aggregates described above, the choice of tax system is important for the behavior of tax-preferred consumption choices reported at the bottom of Table 3. Since tax deductions are modeled explicitly only under the ITC, the reduction in statutory tax rates in this tax system diminishes the value of itemized deductions for charitable giving, home-mortgage interest, and local property taxes. Some households therefore face a substitution effect that acts as a disincentive for charity²² and homeownership²³. This effect is evident under ITC where charitable giving, owner-occupied housing and homeownership are all reduced, but not under the BTF where each are increased due to the sole presence of the income effect. The reduction in the stock of owner-occupied housing under the ITC occurs particularly from young households delaying the purchase of a home. As choosing to rent eliminates the ability of a household to use owner-occupied housing as collateral for borrowing and as a tax-deferred saving vehicle, there is relatively larger increase in the stock of financial assets under the ITC.

The choice of tax system is also important for changes to (productivity-unweighted) market labor hours. Despite similarly increased effective labor supply, hours fall slightly under the ITC while they increase under the BTF. This result indicates that the net employment change under the ITC is composed of relatively more hours from high-productivity workers and relatively less hours from low-productivity workers, whereas the net employment increase under the BTF is relatively balanced across productivity types. This difference, illustrated in Figure 2, is largely due to the labor choices of single workers. Under the BTF (right), single workers of productivity types two through four choose similarly large changes from part-time work into both full-time work and unemployment, the latter of which occurs as early retirement. While there is comparable movement into full-time work under the ITC (left), there is no policy-induced early retirement. Additional important differences are observed among single workers of the fourth productivity type and married workers of the first productivity type, where workers who change employment status choose less hours prior to retirement under the ITC. Indicative of interaction between tax-preferred consumption and labor hours, these household are among those who faced a policy-induced delay in home purchase and have thus accumulated a relatively larger stock of financial assets from which to draw down upon for non-housing consumption.

Table 4 reports policy-induced welfare changes in terms of the percent change in life-time consumption in the alternative steady state that would be needed to make households indifferent to being born in the initial steady state. At the aggregate level both the ITC

²²See Andreoni and Paine (2013) for a survey of the price effects on charitable giving.

²³Hilber and Turner (2014) find that the home-mortgage interest deductions promotes homeownership for high-income households in regions with an elastic supply of housing. Gervais (2002) and Cho and Francis (2011) find that removal of the deduction also reduces the stock of owner-occupied housing.

and the BTF show similar welfare improvements, with respective consumption equivalent variation (CEV) of -0.7% and -0.8% . While similar, differences in CEV by productivity type do show some variation across tax systems due to explicit tax detail incorporated under the ITC. In particular, the negligible welfare change among the lowest two labor-productivity type single households under the ITC reflects that the explicit modeling of the standard deduction leaves these households with little or no taxable income, rendering them relatively unaffected by this policy change. Similarly, the difference in welfare improvement among the fourth productivity type single household reflects interaction between the reduced value of deductions and labor supply behavior under the ITC, which the BTF fails to capture.

4.2.2 Transition Comparison

Economic activity over the first decade of the debt-financed rate-cut can be observed across tax systems in Figure 3. Following the initially larger increase in effective labor supply under the ITC, due in part to relatively strong response from high-productivity workers, firms observe a higher marginal product of capital and respond by increasing their use of capital and producing more output. The accumulation of capital is financed by an increase in households' financial assets at the expense of market and housing consumption. While the stock of housing service consumption follows a similar pattern across tax systems, the relatively larger decrease in owner-occupied housing under the ITC reflects this tax system capturing the reduced value of deductions, which causes households to hold relatively more financial assets. As this results in relatively larger taxable income bases under the ITC, the federal tax revenue loss is relatively smaller and there is less debt accumulation over the first decade.

Over the transition path, the increase in effective labor supply declines while labor hours remain high under the ITC, indicating a relatively larger concentration of less-productive workers over time under this tax system. This occurs as the initial fall and subsequent increase in the after-tax wage rate are relatively larger under the ITC due to the respective magnitude of changes to effective labor and business capital over the first decade. While this pattern induces high-productivity workers cut back on hours from a relatively stronger income effect, it induces and low-productivity workers to increase labor hours from a relatively stronger substitution effect.

The rank-order differences in aggregate responses across tax systems over the first decade have substantial quantitative implications. For example, two years after the policy change, the ITC projects that nominal GDP will be \$136 billion higher than that projected by the BTF; this projected difference grows to \$152 billion by the fifth year. Similarly, the differential in labor hours amount to 2.1 billion more potential hours under the ITC on average each year over the first decade. On the fiscal side, the ITC projects that federal revenues will be \$132 billion higher than projected by BTF over first decade. This implies that the projected level of federal debt held by the public at the end of the 10-year window is \$336 billion lower under the ITC.²⁴

²⁴These figures are computed relative to the Congressional Budget Office's baseline forecast presented in *The Budget and Economic Outlook: 2018 to 2018* under the assumption that the path of inflation is unaffected by the policy change.

4.3 Policy Experiment 2: Expansion of EITC

We simulate a permanent expansion of the EITC that increases the maximum credit and changes the phase-in and phase-out regions for low-income, childless households. In particular, the maximum credit is increased from its 2018 value of \$519 to \$2,000 for both single and married childless households. The credit, which remains fully refundable, is completely phased-in for households of either family composition at a labor income level of \$10,180. It begins to phase out at labor income levels of \$16,250 and \$21,930 for single and married households respectively, becoming completely phased-out at \$32,750 and \$38,430. The expanded schedule is shown graphically in Figure 4, juxtaposed against the present law schedule for childless households and households with one child.

Single individuals account for most of the conventional revenue effect both because they are more likely to be childless than married couples and because they fall within the income range to qualify for the expanded credit. Under the present-law schedule, the lowest-productivity single workers are the primary credit recipients. Under the alternative-law schedule, holding constant initial labor income, the lowest three productivity types would largely qualify for the credit with the fourth productivity type qualifying only at younger and older ages due to lifecycle patterns of dependents and labor productivity. While the lowest productivity singles would be located along the phase-in region of the alternative-law schedule, the second through fourth productivity types would be on the flat or phase-out region of the schedule and further from the maximum credit region as their income rises. This initial positioning around the alternative-law schedule creates an incentive for these individuals to change their labor hours and hence labor income towards the maximum credit region.²⁵

4.3.1 Steady State Comparison

Changes to select aggregates across steady states are reported in the right column of Table 3 for both tax systems. As with the rate reduction simulation, each tax system produces similar changes to macroeconomic activity in the alternative steady state. In each case, output, business capital, and effective labor supply all marginally decrease. However, similarly small decreases in effective labor supply coinciding with large differences in market labor hours imply substantial variation in demographic responses across tax systems. Under the ITC, the increase in labor hours of 0.4% is indicative of a substantial increase in labor hours by low-productivity workers outweighing a decrease in labor hours of middle-productivity workers to qualify for the expanded credit. This contrasts with the response under the BTF, where the net change in labor hours of approximately zero reflects a roughly equal increase in hours by low-productivity individuals and decrease in hours of middle-productivity individuals. This negative net change in hours is inconsistent with the general findings of the literature, which suggests a positive net hours response when considering all margins of labor adjustment simultaneously.²⁶ Under both tax systems, the small increase in housing and market consumption is driven by low-productivity single workers who are generally working more. The relatively larger increase in consumption and decrease in federal tax revenue under ITC, however, reflects

²⁵It has been documented that the Earned Income Tax Credit is associated with substantial overclaims (IRS, 2014) and incomplete take-up (Plueger, 2009), which in dollar terms have offsetting effects on total credit outlays. Nonetheless, we assume full-compliance and take-up in our simulations so that the ITC and BTF tax systems can be analyzed in a comparable fashion.

²⁶See Nichols and Rothstein (2016) for a recent survey of the literature on labor supply and the EITC.

a large increase in labor hours of these workers who take up the expanded credit.

The important implication of the tax system choice for labor hours is shown in Figure 5, where we decompose the employment response among single workers across productivity types for the ITC (top) and the BTF (bottom). Consistent with Eissa and Liebman (1996), we find that for both tax systems single workers exhibit extensive margin movement into the labor force. Along the intensive margin, however, we find that under the BTF net movement out of full-time work dominates net movement into full-time work, leaving hours among singles approximately unchanged when combined with the extensive margin response. The ITC instead exhibits a response where the movement into full-time work dominates along the intensive margin, increasing hours overall in a manner consistent with the findings of Chetty et al. (2013) especially among the lowest productivity workers. While high-productivity singles show almost no change in employment under the ITC, their employment response is substantial under the BTF. Since this group earns labor income well above the credit eligibility threshold, this response is due to the inability of the BTF to target the specific households directly affected by this policy change.

We report the CEV in Table 5 across steady states. At the aggregate level, the ITC tax system exhibits an average welfare improvement smaller than the BTF tax system, with CEV of -0.2% and -0.5% respectively. Consistent with the notion that this particular policy change leaves most married households unaffected directly, we observe that married households have welfare changes that are substantially below that of single households. As the largest beneficiaries of the expanded credit are the lowest two productivity-type singles for both tax systems, we observe large welfare improvements among this demographic. The difference among the third productivity-type single households is the primary source of aggregate CEV differences, as the welfare improvement for the BTF is substantially larger than that of the ITC. Since the measure of third-productivity singles who have initial income levels within the eligibility threshold is small, this result reflects the inability of the BTF to capture the truncated nature of this policy with a smooth approximation. Relative to the ITC, the result is an overstatement of welfare improvement sufficiently strong to affect the CEV at the aggregate level.

4.3.2 Transition Comparison

Aggregate output increases over the first five years following the debt-financed expansion of the earned income tax credit as shown in Figure 6. The extent to which it is driven by the labor supply behavior of households differs qualitatively and quantitatively across tax systems: Households respond to the credit expansion by increasing labor hours by 1.1% and 0.1% on average over the first five years under the ITC and BTF respectively. This variation across tax systems is nearly dichotomous, as the empirical evidence surveyed by Nichols and Rothstein (2016) suggests that the EITC has nonnegligible, positive effect on overall employment as exhibited under the ITC. The relatively larger tax bases that result under the ITC generate a relatively smaller federal revenue loss and accumulation of public debt over the first half of the decade.

Over the second half of the first decade there is a shift in rank order of aggregate responses across tax systems. Despite the increase in employment from the lowest productivity, middle productivity workers reduce hours to receive the expanded credit, and the tax base shrinks as credit payments increase, increasing the accumulation of public debt over the second half of the decade. Cumulative deficits begin to crowd out private capital accordingly under each tax system over the second five years. Firms in both tax

systems respond to the rising cost of capital by using less in production over this period.

The importance of using an explicit tax system for modeling targeted policy changes such as this is further reflected in the quantitative projections for the years following implementation. We find 1.5 billion more potential labor hours on average each year for the first decade under the ITC than the BTF. While this is attributed to lowest productivity workers increasing employment more under the ITC, there are other incentives created for middle productivity workers. As the gross wage rate falls over time, some workers decrease employment to receive the credit, or a larger credit, and EITC payouts increase. As a result of more workers adjusting labor hours to take advantage of the expanded EITC, the federal tax revenue differential between the two systems grows to \$24 billion over the first decade. The implied level of federal debt held by the public projected by the ITC is \$241 billion larger than under the BTF by the end of the first decade.

5 Conclusion

This paper has examined the extent to which a lack of explicit tax detail in heterogeneous-agent models has macroeconomic implications relevant for tax policy analysis. We have incorporated within a large-scale OLG model an internal tax calculator that explicitly models key provisions in the Internal Revenue Code and conditions on idiosyncratic household characteristics when determining labor income tax liabilities. Using this tax system and an unconditional smooth tax function in turn to simulate policy changes, we have demonstrated that the explicit modeling of tax policy allows households to adjust their behavior and react optimally to the policy change as specified by the proposal. In particular, for the statutory tax rate reduction analyzed here, households adjust their tax-preferred consumption behavior in response to the changed value of deductions. Similarly, for the EITC expansion, only households within the expanded credit schedule adjust hours to receive the larger credit. Neither of these behavioral responses were observed in simulations where a smooth tax function was used in lieu of the internal tax calculator.

In the steady state analyses, the policy-induced behavioral responses resulting from the explicit tax policy modeling under the internal tax calculator were unimportant for changes to macroeconomic aggregates. Along the transition path immediately following both policy changes, however, this behavior was associated with a different time path of labor supply, private capital and public debt accumulation, generating quantitative and qualitative differences in macroeconomic aggregates across tax systems. These findings indicate that while the use of unconditional smooth tax functions may be appropriate for steady-state analysis within heterogeneous-agent models, the inclusion of explicit tax detail is important for a reliable transition analysis of tax policy changes.

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7 Tables and Figures

Table 1: Financial Intermediary Balance Sheet and Income Statement

Assets	Liabilities	Income	Expenses
	D_{t+1}		$r_t^p D_t$
K_{t+1}		$r_t K_t$	$\delta^K K_t + \Xi_t$
H_t^r		$p_t^r H_t^r$	$\delta^r H_{t-1}^r$
B_{t+1}		$\rho_t B_t$	

Table 2: Absolute Errors for Aggregate Tax Rates Changes on Labor Income

	ITC	BTF	ITC	BTF
	Rate Reduction		EITC Expansion	
Average Tax Rate Error (%)	0.15	0.09	0.13	0.13
Effective Marginal Tax Rate Error (%)	0.13	0.01	0.06	0.00

All errors expressed as absolute percentage point differences from ITM rate change

Table 3: Steady State Comparison

	ITC	BTF	ITC	BTF
	Rate Reduction		EITC Expansion	
Output	1.2	1.3	-0.1	-0.1
Business Capital Stock	1.7	1.7	-0.0	-0.1
Effective Labor Supply	1.0	1.1	-0.2	-0.1
Labor Hours	-0.1	1.1	0.4	0.0
Market Consumption	1.7	1.7	0.1	0.0
Housing Service Consumption	1.6	1.8	0.2	0.1
Federal Tax Revenue	-3.0	-2.9	-1.1	-0.9
Household Financial Deposits	3.2	1.2	0.0	0.0
Owner-Occupied Housing Stock	-0.5	2.0	0.1	0.1
Homeownership Ratio (p.p.)	-0.2	0.1	0.0	0.1
Charitable Giving	-0.1	1.8	0.3	0.1

All variables with the exception of the homeownership ratio are expressed as percent differences across steady states

Table 4: CEV (%) for Rate Reduction

	ITC	BTF	ITC	BTF
Productivity	Single Households		Married Households	
1	-0.1	-0.2	-0.2	-0.2
2	-0.0	-0.3	-0.6	-0.6
3	-0.5	-0.5	-0.9	-0.8
4	-1.1	-0.7	-1.3	-1.2
5	-1.6	-1.5	-2.4	-2.2
	ITC	BTF		
Aggregate	-0.7	-0.8		

Table 5: CEV (%) for EITC Expansion

	ITC	BTF	ITC	BTF
Productivity	Single Households		Married Households	
1	-1.3	-0.9	-0.2	-0.3
2	-1.7	-2.0	0.1	-0.0
3	-0.1	-0.5	-0.1	-0.0
4	-0.0	-0.1	-0.1	-0.0
5	-0.1	-0.1	-0.1	-0.0
	ITC	BTF		
Aggregate	-0.2	-0.5		

Figure 1: Labor Income and Tax Liabilities: Internal Tax Calculator vs. Tax Function

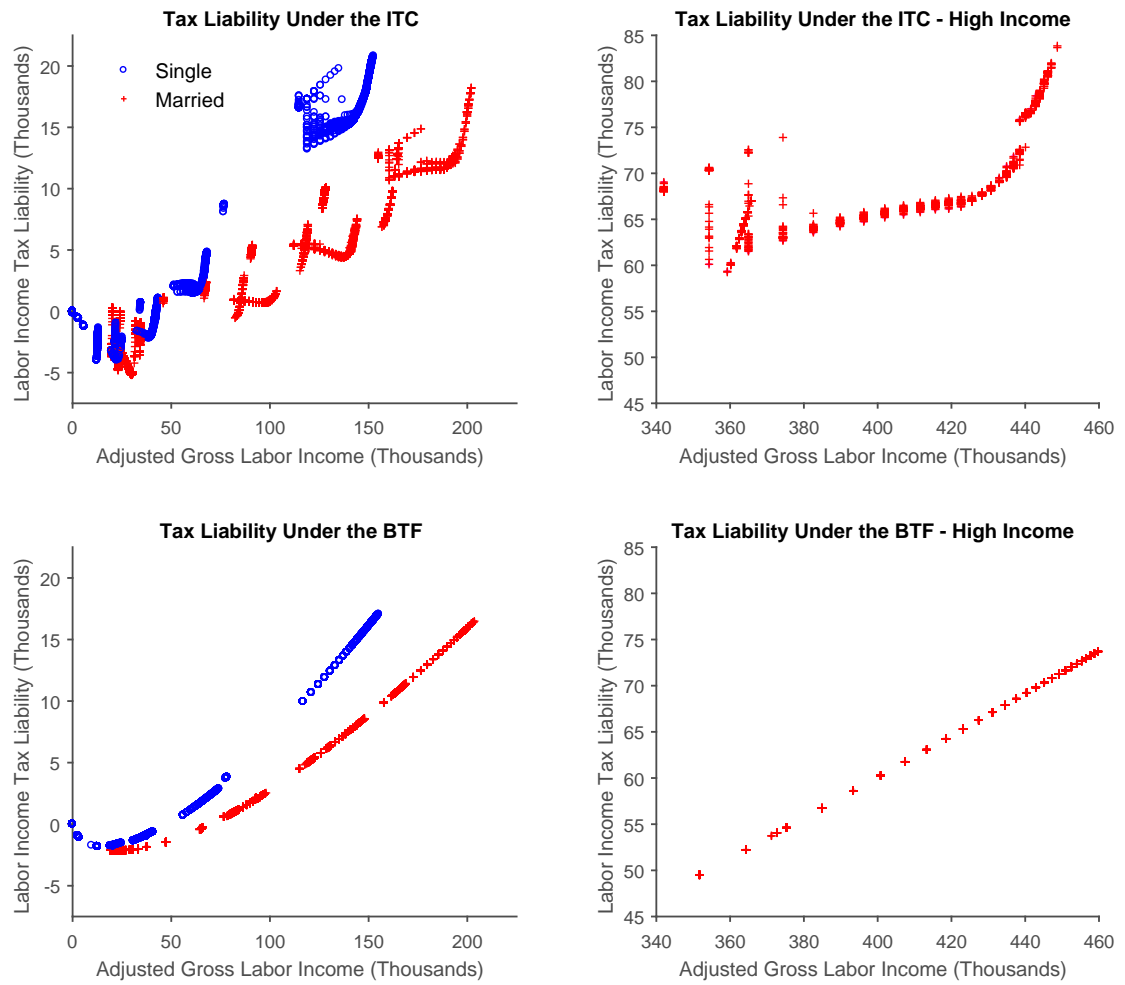


Figure 2: Rate Reduction: Percentage Point Change in Employment Status by Productivity Type and Family Composition Across Steady States

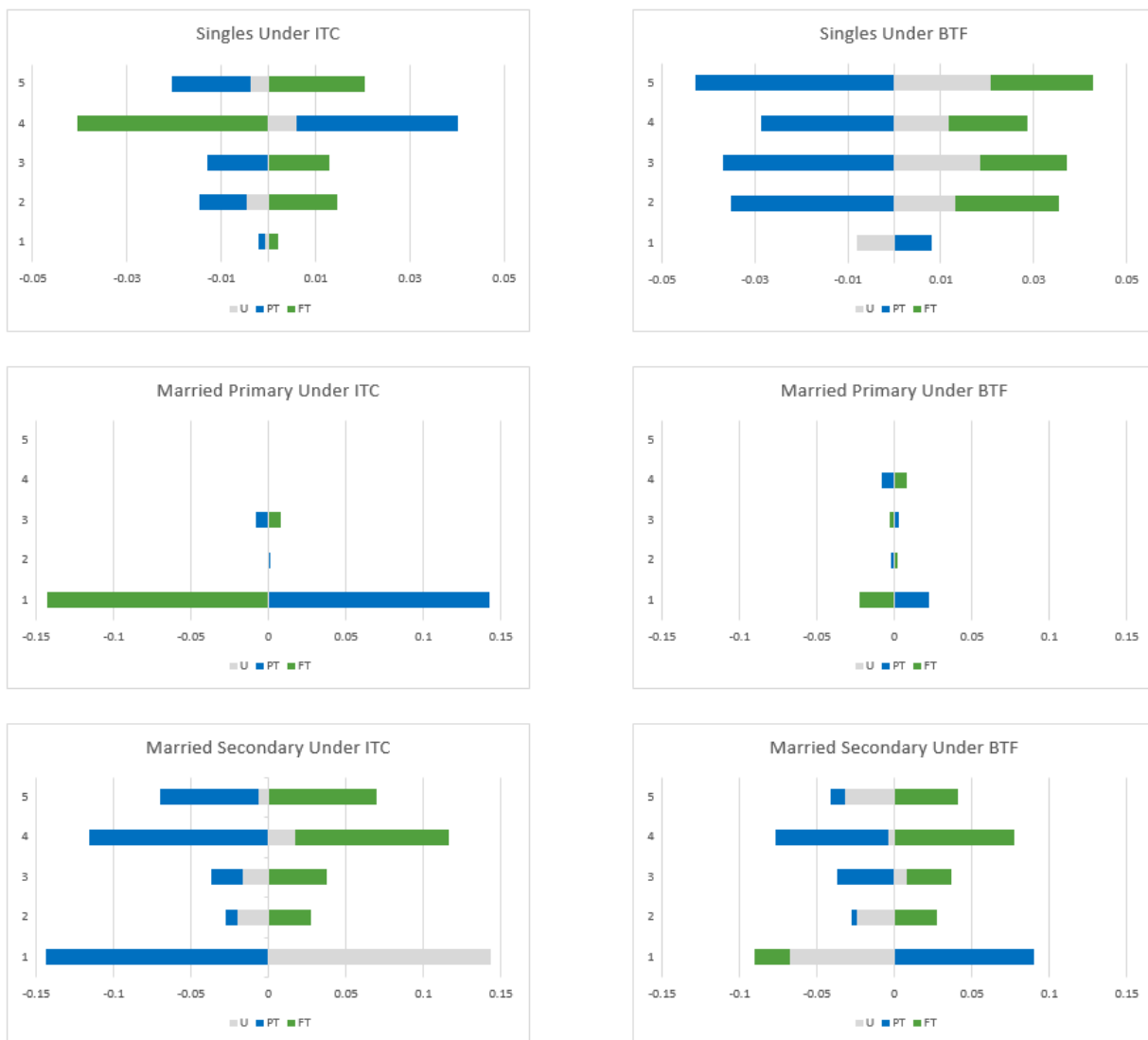


Figure 3: Rate Cut: Aggregates During Transition Relative to Initial Steady State

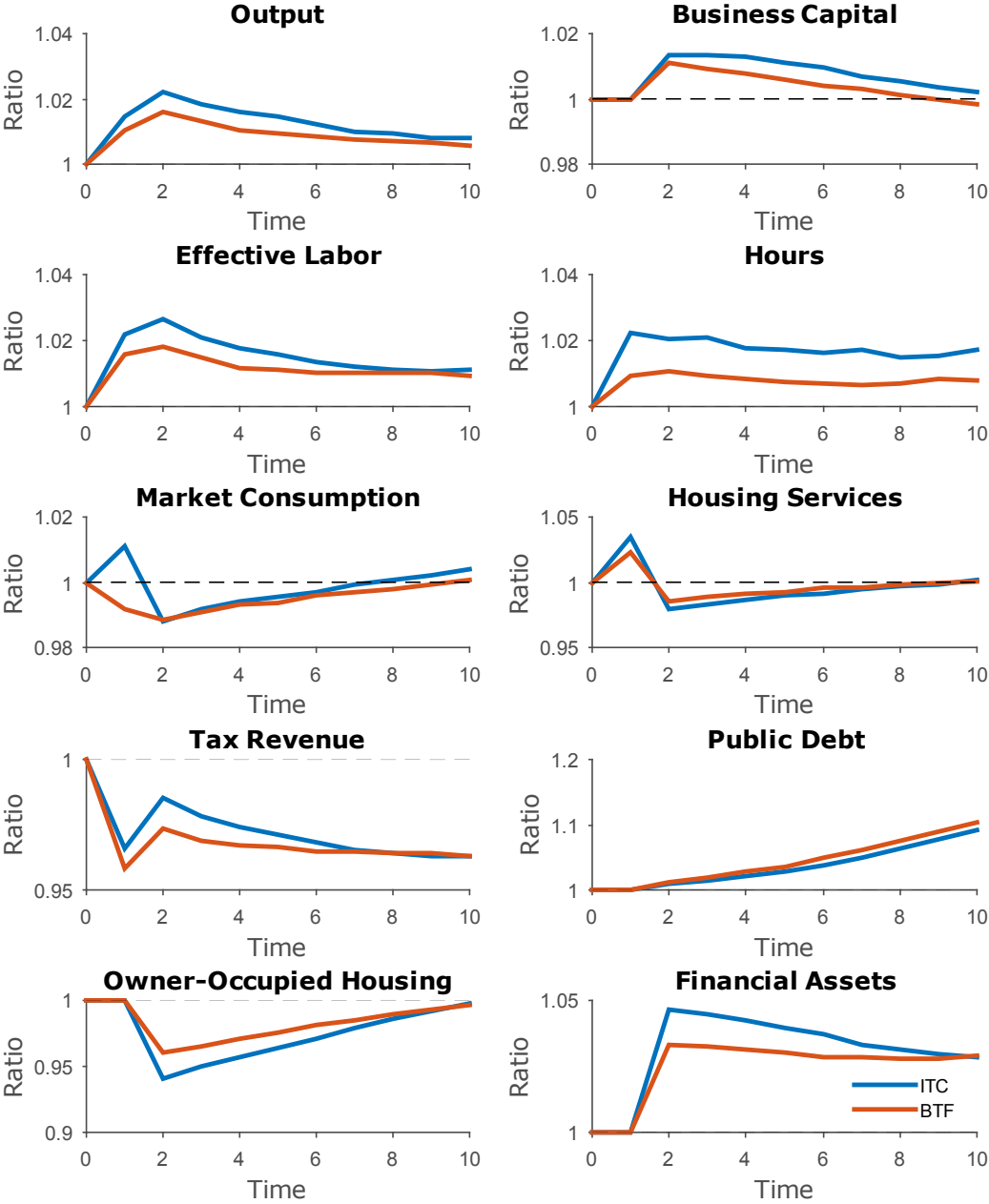


Figure 4: Policy Experiment: *EITC Expansion*

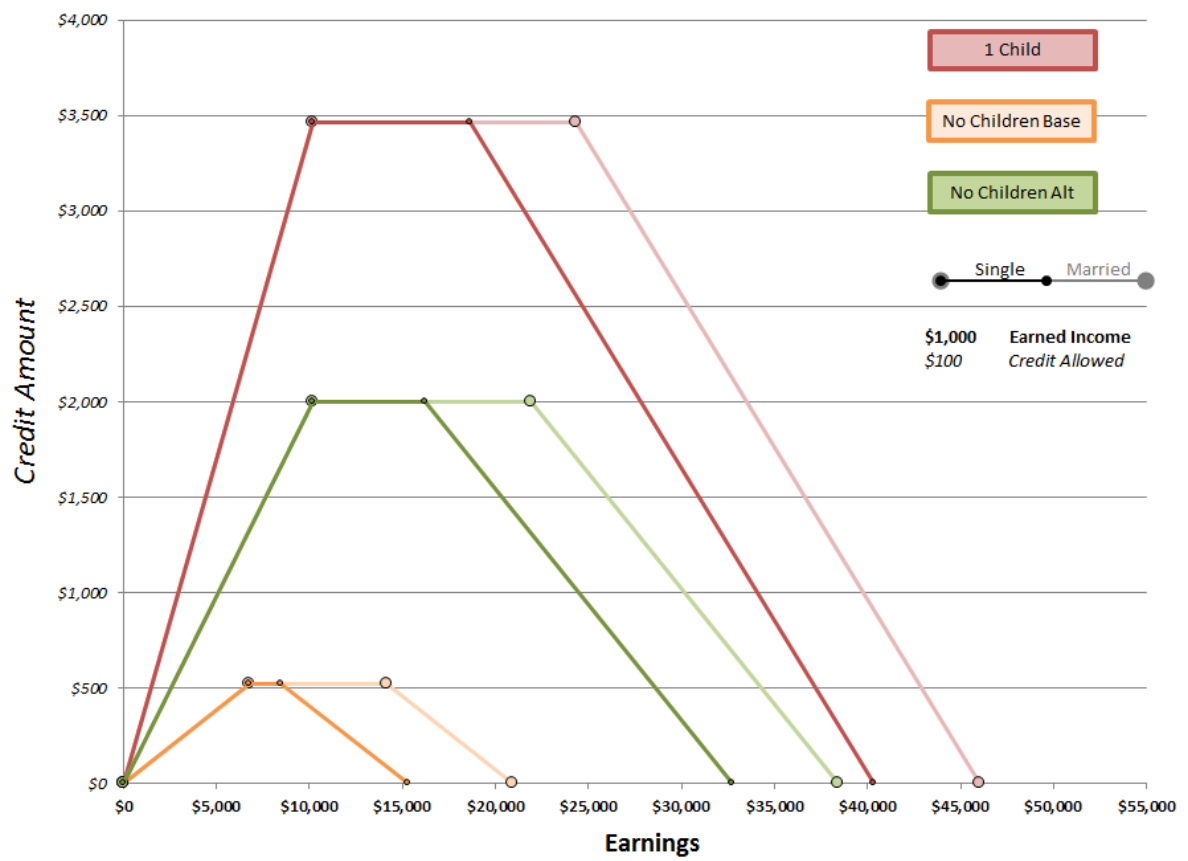


Figure 5: EITC Expansion: Percentage Point Change in Employment Status by Productivity Type for Single Households Across Steady States

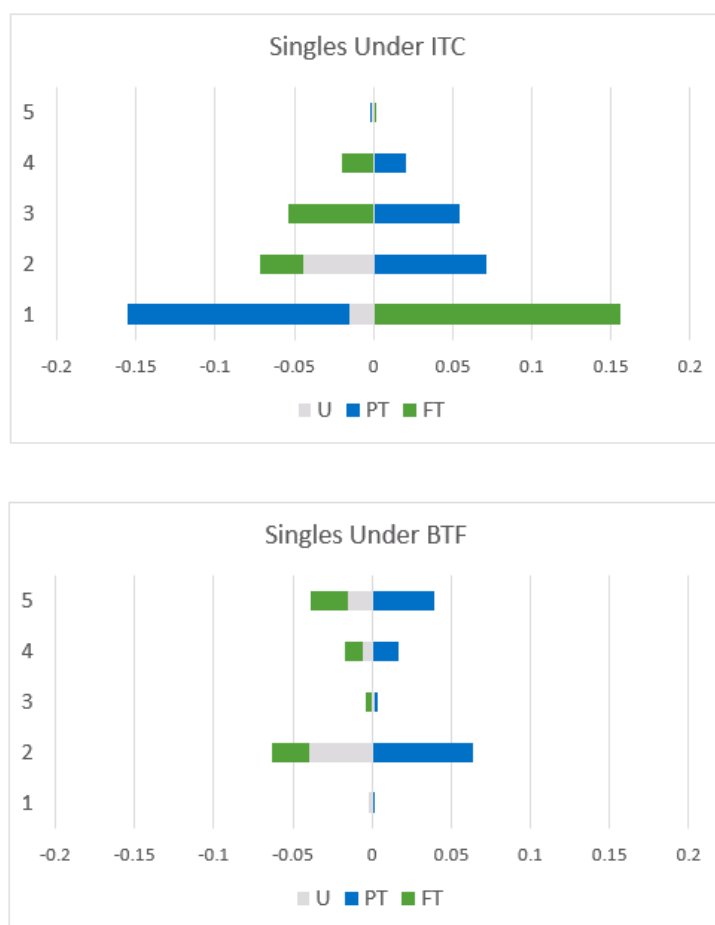
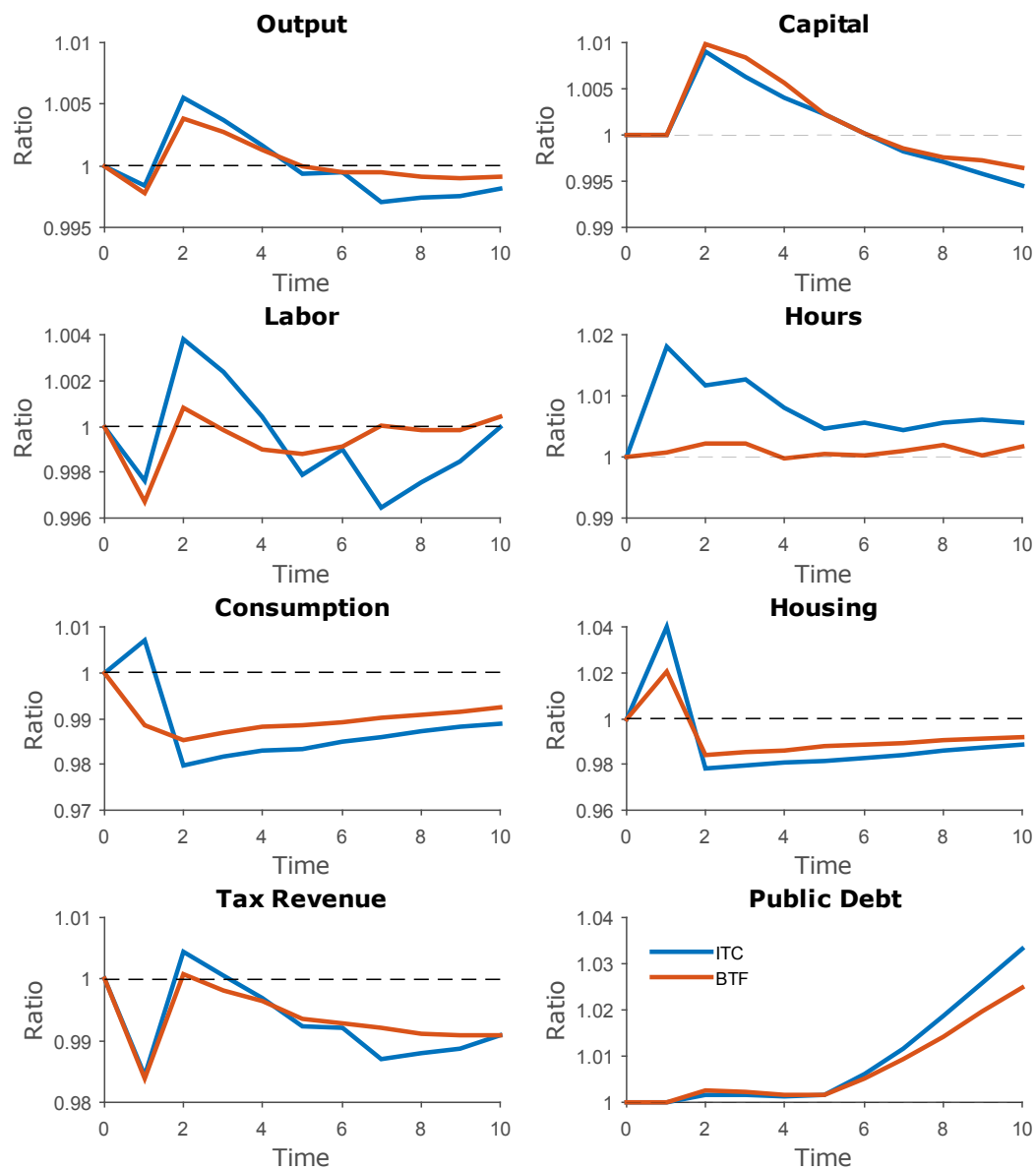


Figure 6: EITC Expansion: Aggregates During Transition Relative to Initial Steady State



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A Calibration

A.1 Non-Tax Policy Parameter Values and Targets

A.1.1 Demographics

Discrete time in the model passes at an annual frequency. At the end of each period, all individuals aged $j = R = 40$ retire and begin to face mortality risk while those aged $j = J = 66$ die with certainty. Model age $j = 1$ is set to correspond with actual age 25 so that model age $j = 40$ corresponds with actual age 64, and model age $j = 66$ corresponds with actual age 90.

The population growth rate is set to $v_P = 0.0075$, which is the average annual U.S. population growth rate projected by the Census Bureau for years 2018-2028. The rate of technological progress is set so that the steady state growth rate of aggregate output is equivalent to the average annual real GDP growth rate of 1.93% projected by the Congressional Budget Office¹ (CBO) for years 2018-2028. Since aggregate output grows at rate $v_P + v_A$ in a macroeconomic steady state, we set $v_A = 0.0118$.

Conditional survival probabilities π_j are set to $\pi_j = 1$ for ages $j < R$, as households in the model only begin to face mortality risk upon retirement. We calibrate the latter conditional survival probabilities using the Social Security Administration's *2013 Actuarial Life Table*. We compute a weighted average over males and females for each given age to obtain a single age-variant series which we apply to all households.

The measure of households $\Omega_j^{z,f}$ is constructed in four steps.² First, given the constant rate of population growth and time-invariant mortality, we construct a stationary age profile of households as $\Omega_{j+1} = (\Omega_j \pi_j) / \Upsilon^P$. Next, the joint density Ω_j^f is constructed directly by computing the shares of joint and non-joint tax units out of total units over each age 25 through 90 projected for 2018 using the ITM. Third, the density Ω^z is constructed independently of age and family composition under the assumption that the mass of each age - family composition group is equivalent for each of the nz productivity types, which implies $\Omega^z = 1/nz \forall z \in \mathbb{Z}$.³ Finally, $\Omega_j^{z,f} = \Omega_j^f \Omega^z$ is normalized to have the property:

$$\int_{\mathbb{J}} \int_{\mathbb{Z}} \sum_{f=s,m} \Omega_j^{z,f} = 1$$

A.1.2 Firm Production Technology and Housing

To calculate the share of output in production for private and public capital respectively, we borrow from the method of Cooley and Prescott (1995) which allocates the ambiguous components of aggregate income in proportion to each factor's share in measured output.^{4,5} Using data from the National Income and Product Accounts (NIPA) of the

¹Unless otherwise indicated, projections from the Congressional Budget Office are from *The Budget and Economic Outlook: 2018 to 2028*, April 2018

²The time subscript is omitted for a cleaner exposition.

³This is a natural assumption because, as described in Section A.1.3, productivity groups will be taken to represent lifetime labor income quintiles.

⁴While Cooley and Prescott (1995) calculate factor income share of GNP, we follow their methodology to instead calculate factor income shares in GDP.

⁵We consider ambiguous components to be proprietor's income, the statistical discrepancy, taxes on production and imports, and the current surplus of government enterprises less subsidies. While the latter

Bureau of Economic Analysis for 2007-2016, we first calculate 36.17% as the joint share of GDP for both public and private non-residential capital. We then repeat the calculation private non-residential capital only, we yields private capital's share of production $\alpha = 0.3265$. Finally, we take public capital's share of production to be the residual of the joint share and private capital's share so that $g = 0.0352$.

As in Fernandez-Villaverde and Krueger (2010), rates of economic depreciation on productive capital and housing capital are calculated using the steady state expression for the investment to capital ratio. For productive private capital this expression is $I^K/K = (\Upsilon_A \Upsilon_P - 1 + \delta^K)$. Solving for the depreciation rate yields $\delta^K = ((I^K/K) - \Upsilon_A \Upsilon_P + 1)$. Using the average annual investment flows and stocks of private non-residential capital as reported by NIPA for years 2007-2016 yields $\delta^K = 0.0799$. Using the analogous expression for public capital, which we take here to include federal, state, and local capital, we compute $\delta^G = 0.0317$ over the same period. For housing capital, we calculate the depreciation rates using the average annual investment flows and stocks of private residential capital and consumer durables as reported by NIPA for years 2007-2016. The expression for the owner-occupied housing capital depreciation is the same as that for private capital, which obtains $\delta^o = 0.0555$. The depreciation rate on rental housing capital differs from owner-occupied housing capital because rental housing investment is assumed to be usable contemporaneously with investment flows. For rental housing capital, the investment to capital ratio is $I^r/H^r = (\Upsilon_A \Upsilon_P - 1 + \delta^r)/(\Upsilon_A \Upsilon_P)$, which has the associated depreciation rate $\delta^r = \Upsilon_A \Upsilon_P ((I^r/H^r) - 1) = 0.0570$.

The financial intermediary faces quadratic costs of capital adjustment equal to $\Xi_t = \frac{\xi^K}{2} (\frac{K_{t+1}}{K_t} - \Upsilon_P \Upsilon_A)^2 K_t$. Given the rates of population growth and technological progress, this adjustment cost function is parameterized by ξ^K , which for purposes of the simulations is set to 6.

Following Gervais (2002), Fernandez-Villaverde and Krueger (2010), and Cho and Francis (2011), we set the minimum down-payment required on owner-occupied home purchases to $\gamma = 0.20$. This figure also closely corresponds to the median loan-to-value ratio of 77% for owner-occupied housing units manufactured between 2010-2015 as reported in the Census Bureau's 2015 *American Housing Survey*. Furthermore, as in Gruber and Martin (2003), we assume transaction costs associated with changing your housing status unit are symmetric for buying and selling. While the authors find median costs of 7% and 2.5% of housing value for selling and purchasing respectively, we conservatively choose the midpoint value so that $\phi^o = \phi^r = 0.05$.

To restrict households from purchasing unfeasibly small residences, we assume there is a lower bound on the support of rental housing \underline{h}^r and owner-occupied housing \underline{h}^o such that $0 < \underline{h}^r \leq \underline{h}^o$. The lower bound on owner-occupied housing value is set to target a homeownership ratio of 0.637 as reported for 2015 by the *American Housing Survey*. Reflecting the ratio of housing to food spending for the lowest decile of households in 2016 as reported by the Bureau of Labor Statistics (BLS) in the *Consumer Expenditure Survey*, the lower bound on rental housing value is set to be 2.525 times the average minimum consumption levels across family composition.

three are often excluded when calculating factor income shares, we include them so that the aggregate income-output equivalence in the model implies a level of output consistent with measured GDP.

A.1.3 Household Characteristics and Preferences

The rate at which households discount future utility is set to target the observed private business investment to GDP ratio of 0.162 as calculated from NIPA for 2016. We set the subjective discount factor as $\beta = 0.94$ to target this figure under both tax systems.

Individual labor productivity $z_j^{z,f}$ is assumed to consist of two independent components: (i) an age-varying component z_j and (ii) an age-invariant component that depends both on productivity type and family composition $z^{z,f}$, so that $z_j^{z,f} \equiv z_j z^{z,f}$. We let the number of productivity types $n_z = 5$ so that each type z approximates average annual household labor income quintiles. Since a household of productivity type z remains that type over their lifetime, our calibration is consistent with the notion of ‘lifetime’ labor earnings quintiles. This leads to a natural sorting of households by their labor income characteristics, where we define labor income to be the sum of a NIPA-comparable wage income concept and a $(1 - \alpha - g)$ share of pass-through business income.^{6,7}

To calibrate the age-varying component of labor productivity, we adopt the smoothed numerical wage profiles estimated in Rupert and Zanella (2015) for all individuals, normalizing the mean to unity. For the age-invariant component of labor productivity, we set $z^{z,f}$ for $z = \{1, \dots, 5\}$ and $f = \{s, m\}$ endogenously so that average annual labor income over ages $j = \{1, \dots, R\}$ in the model matches the ITM’s 2018 extrapolated values of average annual labor income over ages 25-64 for each respective group, when in present-law baseline equilibrium.⁸ While both potential workers in married households face the same individual labor productivity term $z_j^{z,f}$, there is an exogenous productivity wedge μ^z between primary and secondary workers. This wedge is taken to represent the hourly earnings of secondary workers relative to primary workers, and is computed using the total wage and business income quintiles of primary filers and their spouses as reported by the Medical Expenditures Panel Survey (MEPS) for 2015.

Individual labor supply n is indivisible, and is assumed to correspond with either full-time employment n^{FT} , part-time employment n^{PT} , or unemployment. Following the BLS, we consider full-time employment to correspond with at least 35 weekly hours of work and part-time employment to correspond with at least 1 but no more than 34 weekly hours of work. Using data from the BLS, we find that in 2016 the median full-time and part-time employees work 44.4 and 22.4 hours per week respectively, which are equivalent to about 2310 and 1163 working hours per year. As the BLS reports in the 2016 *American Time Use Survey* that the average individual spends about 3208.4 hours sleeping per year, full-time and part-time work correspond with 41.6% and 21.0% of waking hours spent working. We therefore normalize individual total time endowments to unity, and set $n^{FT} = 0.416$ and $n^{PT} = 0.210$.

The labor disutility coefficients $\psi^s, \psi^{m,1}, \psi^{m,2}$, and fixed cost of employment parameters ϕ^s, ϕ^m , are set so that model-generated lifecycle labor supply approximates the

⁶The BEA does not report distributional characteristics of NIPA wage income. To approximate the desired NIPA wage income quintiles, we derive a ‘NIPA-comparable’ measure using the ITM by adding to AGI wage income (i) combat pay, (ii) employers’ share of the FICA tax, (iii) deferred 401k compensation, (iv) employers share of 401k compensation, (v) employer provided dependent care, (vi) employer health-insurance compensation, (vii) employer HSA compensation, and (viii) employer life-insurance compensation. See Ledbetter (2007) for a comparison of AGI income and NIPA income.

⁷The pass-through business income considered here includes income flows from sole proprietorship, partnerships, S-corporations, estates and trusts, and rents and royalties.

⁸To match the demographics explicitly modeled, we restrict our attention to households of these ages who are not receiving retirement income, e.g. 401(k), IRA, pension, or social security income.

distribution of employment status by earner type — single, married primary, or married secondary — as observed in the MEPS for 2015 and reported in Table A1.⁹ The fixed cost of employment parameter largely determines the workforce participation rate. The extensive margin elasticity, which determines the sensitivity of workers moving into or out of employment in response to changes in the after-tax wage rate, is determined endogenously by the distribution of reservation wages which itself partially depends on the fixed costs associated with working.

The intensive margin elasticity, which determines the sensitivity of workers choosing to supply more or less hours of work in response to changes in the after-tax wage rate conditional on already being employed, depends on the utility function curvature parameters $\zeta^s, \zeta^{m,1}, \zeta^{m,2}$ only in models of continuous labor choice. In models of discrete labor choice such as the one used here, Chang et al. (2011) show that these parameters are largely unrelated to the intensive margin elasticity. Rogerson and Wallenius (2009) and Keane and Rogerson (2012) show that higher values of these parameters imply that aggregate employment fluctuations depend more heavily on changes in the duration of working life, rather than changes in hours worked while employed. Estimated elasticities for secondary earners from their model simulation are systematically higher despite having the same curvature parameter as the primary earner. Perhaps most importantly for calibration purposes, movement along the intensive margin should account for one-third of aggregate employment fluctuations on average over 1970-2009, see Fiorito and Zanella (2012). Taking this into consideration, we choose the same relatively high uniform value for workers and set $\zeta^s = \zeta^{m,1} = \zeta^{m,2} = 5$.

The amount of hours spent on home production have a fixed, inverse relationship to the amount of labor hours. We used the 2016 *American Time Use Survey*¹⁰ to find the average housework hours¹¹ for full time, part time, and unemployed individuals. Full time workers reported spending 0.62 hours per day, part time workers reported 1.52 hours per day, and unemployed individuals reported 1.73 hours. Normalizing available (non-sleep) time to unity yields the following mapping for home work time as a function of labor hours for singles and each married individual:

$$\mathbb{N} = [0.000, 0.210, 0.416] \rightarrow \mathbb{N}^h = [0.114, 0.100, 0.041]$$

Empirically, the value of home production has been measured by multiplying hours spent on housework by the wage rate of domestic workers, see Bridgman (2016). We take a similar approach to calculating the value of home production as a function of non-work hours:

⁹MEPS reports the employment status of each individual in a married household, but does not specify who is the primary earner. Rather than erroneously using gender as an indicator of primary or secondary earnings status, we consider the amount of hours worked. If both individuals are unemployed, we consider the primary earner to be the one who is unemployed. If both individuals are working part time, or one is working part time and one is unemployed, we consider the primary earner to be the one employed part time. Lastly, if at least one earner is employed full time, we consider the primary earner to be the one employed full time. BLS consistent definitions of full-time, part-time and unemployed are used in construction of these targets.

¹⁰Sandra L. Hofferth, Sarah M. Flood, and Matthew Sobek. American Time Use Survey Data Extract Builder: Version 2.6. College Park, MD: University of Maryland and Minneapolis, MN: University of Minnesota, 2017. <https://doi.org/10.18128/D060.V2.6>.

¹¹We summed four variables for home work averages: housework, food prep and cleanup, interior maintenance and exterior maintenance. We chose variables that can be reasonably outsourced for pay, and sounded most like chores, rather than hobbies or leisure activities.

$$c^h(n_j^h) = \begin{cases} w_t \bar{z}^{1,s} n_j^h & \text{if } f = s \\ w_t \bar{z}^{1,s} (n_j^{h,1} + n_j^{h,2}) & \text{if } f = m \end{cases}$$

where $w_t \bar{z}^{1,s}$ is the average wage rate for the lowest productivity type single household.

Childcare expenses take the form $\kappa_j^{z,f} \equiv cc^{z,f} \nu_j^{z,f} n_j$, where labor supply is evaluated at the quantity supplied by the secondary worker for married households. Given the average number of dependents under 13 within a household $\nu_j^{z,f}$, we set the childcare cost scale parameter $cc^{z,f}$ exogenously so that childcare expenses on average for each (z, f) demographic when labor supply is evaluated as the employment targets discussed above match those values imputed by the ITM for 2018.

Given the specification of the non-housing consumption composite in equation (2.6), the relative optimal quantities of ordinary consumption and charitable giving can be expressed for those households not itemizing tax deductions as:

$$\frac{c_j^g}{c_j^i} = \left(\frac{1 - \theta^{z,f}}{\theta^{z,f}} \right)$$

Let $(\bar{c}^g/\bar{i})^{z,f}$ denote average charitable giving as a proportion of labor income targets for each (z, f) combination as computed from the ITM for 2018. Then the consumption composite function can be parameterized endogenously by setting $\theta^{z,f}$ such that:

$$\theta^{z,f} = \left(1 + (\bar{c}^g/\bar{i})^{z,f} \frac{\sum_{j=1}^R i_j^{z,f}}{\sum_{j=1}^R c_j^{i;z,f}} \right)^{-1}$$

which implies that in baseline equilibrium the model reproduces charitable giving targets on average for the working-age population. While this structure allows for the model to generate the U-shaped pattern of average household giving over income (List, 2011), the variance in giving among households of a given demographic will be understated.

The elasticity of substitution between housing and non-housing consumption is set to $\eta = 0.487$ as estimated by Li et al. (2016). The non-housing consumption preference parameter σ is set to target the ratio of private business investment to total private investment of 0.465 as calculated from the 2016 NIPA. The lower bound on permissible ordinary consumption levels \underline{c}^i is set to be an arbitrary 5% larger than the maximum amount of home production consumption that may be obtained from choosing unemployment so that this constraint can potentially bind.

A.1.4 Government: Public Capital and Debt

Productive public capital held by the federal government and the state and local government are both set endogenously so that in present-law baseline they exhibit a value of 18.4% and 55.1% respectively of aggregate output. This value reflects an average over 2007-2016 from NIPA. Given this target, and the assumption that public capital remains fixed under proposed law simulations, the level of investment in public capital can be computed from the equation of motion.

The CBO projects that federal debt held by the public less financial assets relative to GDP will grow from 65.7% in fiscal year 2018 to 84.0% in fiscal year 2028. Because of this large projected increase we calibrate federal debt held by the public in the present-law baseline so that it is approximately 74.6% of aggregate output, the average projected

value over fiscal years 2028-2028. Furthermore, the United States Treasury reports in the December 2017 *Treasury Bulletin* that 21.7% of debt held by the public was held by Federal Reserve Banks at the beginning of fiscal year 2018. Since this portion of public debt does not necessarily crowd out private capital, we endogenously set the net stock of government debt, B , in the present-law baseline so that its level relative to aggregate output is approximately 58.4%.

It is also projected by the CBO that net interest payments on federal debt relative to GDP will be growing over fiscal years 2018-2028. We therefore endogenously set the wedge between rate of return to private capital and government debt, ϖ , so that net interest payments relative to output match the average projected value of 2.60% over this time period.

A.2 Endowment Heterogeneity

As shown in Hugget et al. (2011) for models of this class, differences in initial wealth contribute to a substantial portion differences in lifetime wealth. To account for this relationship we introduce variation in endowments of initial financial wealth a_1 over each demographic, which is indexed by $e = \{1, \dots, ne\} \in \mathbb{E}$.

To specify beginning-of-period endowments for households entering the economy at age $j = 1$, we draw ratios of financial wealth to labor income, \mathbf{A}^f , from the following distribution:¹²

$$\sinh^{-1}(\mathbf{A}^f) \sim N(\mu^f, \sigma^{2:f})$$

where μ^f and $\sigma^{2:f}$ are taken to be the sample mean and variance of the inverse hyperbolic sine of financial wealth to labor income ratios for single and married families headed by a 25 year old in the *Survey of Consumer Finances* for 2013, respectively $\bar{x}^f = \{-0.0439, 0.0045\}$ and $s^{2:f} = \{0.7464, 0.6153\}$.¹³ Letting $\bar{i}^{z,f}$ denote the average lifetime labor income targets, and z_j be the age-varying component of individual labor productivity, we take the product $\bar{i}^{z,f} z_1$ as an approximation of average labor income for the youngest cohort of households. Treating single and married households as separate distributions, we take $ne = 40$ random draws of \mathbf{A}^f for each productivity type, we obtain initial endowments of financial wealth:

$$a_1^{e,z,f} = \mathbf{A}^f z_1 \left(\frac{\bar{i}^{z,f}}{\bar{i}^{nz,f}} \right)$$

where the denominator on the right-hand side normalizes initial endowments by the average labor income of the highest productivity type. Furthermore, we take the minimum drawn value of endowments for each labor productivity type z and family composition f as the lower-bound of wealth support for the respective demographic:

¹² We choose the inverse hyperbolic sine transformation to approximate a natural log transformation while allowing for negative values. See Pence (2006) for a discussion.

¹³ We define financial wealth as financial assets (balances of checking accounts, savings accounts, money market mutual accounts, call accounts at brokerages, prepaid cards, certificates of deposits, total directly-held mutual funds, stocks, savings and other bonds, IRAs, thrift accounts, future pensions, cash value of whole life insurance, trusts, annuities, managed investment accounts with equity interest and miscellaneous other financial assets) less debt (credit card balances, education loans, installment loans, loans against pensions and/or life insurance, margin loans and other miscellaneous loans).

$$\underline{y}^{z,f} = \arg \min(a_1^{e,z,f})$$

While this variation in endowment level does not change the dynamic optimization problem, endowment heterogeneity does add an additional layer of aggregation such that for any variable x :

$$x_{t,j}^{z,f} = \int_{\mathbb{E}} x_{t,j}^{z,f,e} \Omega^e de$$

where $\Omega^e = \frac{1}{ne}$ is the measure of endowment level e . Therefore in each year of the simulation, $nf \times nz \times ne$ households enter the model. This level of aggregation is implicit in Appendix B.3.

B Dynamic Programming Problem

B.1 Stationarity

Along a steady-state balanced growth path, aggregate and individual variables will be growing at a constant rate. In order to apply numerical solution techniques for stationary economies we express these variables in trend-stationary form to adjust for the source of growth, which could be either population growth, technological progress, or both.¹⁴ Letting the tilde accent denote a variable transformed to stationary form, Table A4 lists selected growth-adjusted variables.

Consider aggregate labor supply, N , which in a steady state will be growing due to population growth. While aggregate labor supply will therefore be growing at a rate of v_P , the stationary variable \tilde{N} will be constant. Every other aggregate variable will be growing in a steady state due to both population growth and technological progress. Therefore, they can be made stationary by dividing each with the product AP . While each of these non-stationary aggregates will be growing at the rate of $v_P + v_A$ in a steady state, the stationary variable will be constant.

The measure of households' age and productivity, Ω , will be growing at a rate of v_P due solely to population growth, and therefore can be made stationary by dividing by the total population. Every other individual variable grows at a rate of v_A in a steady state due solely to technological progress. These variables can therefore be made stationary by dividing each with the level of technology.

B.2 Stationary Recursive Formulation

We express the model described in Section 2 as a trend stationary dynamic program, which is solved numerically. In doing so, we first perform a change of variables to mitigate the curse-of-dimensionality problem by reducing the two-dimensional household state space to a single dimension: Redefining the household value functions from $V_{t,j}^{z,f}(\tilde{a}_j, \tilde{h}_j^o)$ to $V_{t,j}^{z,f}(\tilde{y}_j)$, where stationary net worth \tilde{y}_j is the sum of the stationary stock of financial assets \tilde{a}_j and owner-occupied housing services \tilde{h}_j^o , we simultaneously expand the set of household choice variables from $\{\tilde{a}_{j+1}, \tilde{h}_{j+1}^o; \tilde{x}_j, n_j, \tilde{c}_j^i, \tilde{c}_j^g, \tilde{h}_j^r\}$ to $\{\tilde{y}_{j+1}; \tilde{x}_j, n_j, \tilde{c}_j^i, \tilde{c}_j^g, \tilde{h}_j^r, \tilde{h}_j^o, \tilde{a}_j\}$. Choosing the optimal value of future net worth under perfect foresight, households know with certainty its composition across financial assets and owner-occupied housing services by definition of the value function. With this information, they may then choose the optimal composition of current net worth across these same choice variables in a time-consistent fashion. Solving the recursive dynamic program by backwards induction as described in Appendix C reflects this structure.

Taking prices, bequests, and government policy as given, households compare the indirect utility generated by decisions associated with owning a home against the welfare generated by decisions associated with renting:

$$\mathbf{V}_{t,j}^{z,f}(\tilde{y}_j) = \max\{\mathbf{V}_{t,j}^{o;z,f}, \mathbf{V}_{t,j}^{r;z,f}\} \quad (\text{B.1})$$

where $\mathbf{V}_{t,j}^{o;z,f}$ and $\mathbf{V}_{t,j}^{r;z,f}$ are the current period value functions associated with owning and renting respectively for a household of demographic (j, z, f) .

¹⁴See King et al. (2002) for a detailed discussion of trend stationarity.

The value function for a household that will be a homeowner in period t is:

$$\mathbf{V}_{t,j}^{o;z,f}(\tilde{y}_j) = \begin{cases} \max_{\tilde{y}_{j+1}; \tilde{x}_j, n_j \in \mathbb{N}} \mathbf{U}_{t,j}^{o;z,s}(\tilde{x}_j, n_j) + \beta \pi_j \mathbf{V}_{t+1,j+1}^{z,s}(\tilde{y}_{j+1}) & \text{if } f = s \\ \max_{\tilde{y}_{j+1}; \tilde{x}_j, n_j^1, n_j^2 \in \mathbb{N}} \mathbf{U}_{t,j}^{o;z,m}(\tilde{x}_j, n_j^1, n_j^2) + \beta \pi_j \mathbf{V}_{t+1,j+1}^{z,m}(\tilde{y}_{j+1}) & \text{if } f = m \end{cases} \quad (\text{B.2})$$

where:

$$\mathbf{U}_{t,j}^{o;z,f} \equiv \begin{cases} \max_{\tilde{c}_j^i, \tilde{c}_j^g, \tilde{h}_j^o} \log(\tilde{x}_j) - \psi^s \frac{n_j^{1+\zeta^s}}{1+\zeta^s} - F^s & \text{if } f = s \\ \max_{\tilde{c}_j^i, \tilde{c}_j^g, \tilde{h}_j^o} \log(\tilde{x}_j) - \psi^{m,1} \frac{(n_j^1)^{1+\zeta^{m,1}}}{1+\zeta^{m,1}} - \psi^{m,2} \frac{(n_j^2)^{1+\zeta^{m,2}}}{1+\zeta^{m,2}} - F^m & \text{if } f = m \end{cases} \quad (\text{B.3})$$

$$\tilde{x}_j \equiv \left(\sigma \tilde{c}_j^\eta + (1 - \sigma)(\tilde{h}_j^o)^\eta \right)^{1/\eta} \quad (\text{B.4})$$

As a subset of the full set of constraints for this optimization problem, the household considering being a homeowner in period t faces:

$$\tilde{c}_j^M + \tilde{c}_j^g + (r_t^p + \delta^o) \tilde{h}_j^o + \tilde{y}_{j+1} \Upsilon_A \leq (1 + r_t^p) \tilde{y}_j + b \tilde{e} q_t + \tilde{i}_{t,j}^{z,f} - \tilde{T}_{t,j}^{z,f} - \tilde{\kappa}_j^{z,f} - \phi^r \tilde{h}_{j+1}^r \Upsilon_A \quad (\text{B.5})$$

$$\tilde{h}_j^o \geq \underline{h}^o \quad (\text{B.6})$$

$$\tilde{y}_j \geq \gamma \tilde{h}_j^o \quad (\text{B.7})$$

$$\tilde{h}_j^r = 0 \quad (\text{B.8})$$

The value function for a household that will be a renter takes the form:

$$\mathbf{V}_{t,j}^{r;z,f}(\tilde{y}_j) = \begin{cases} \max_{\tilde{y}_{j+1}; \tilde{x}_j, n_j \in \mathbb{N}} \mathbf{U}_{t,j}^{r;z,s}(\tilde{x}_j, n_j) + \beta \pi_j \mathbf{V}_{t+1,j+1}^{z,s}(\tilde{y}_{j+1}) & \text{if } f = s \\ \max_{\tilde{y}_{j+1}; \tilde{x}_j, n_j^1, n_j^2 \in \mathbb{N}} \mathbf{U}_{t,j}^{r;z,m}(\tilde{x}_j, n_j^1, n_j^2) + \beta \pi_j \mathbf{V}_{t+1,j+1}^{z,m}(\tilde{y}_{j+1}) & \text{if } f = m \end{cases} \quad (\text{B.9})$$

where:

$$\mathbf{U}_{t,j}^{r;z,f} \equiv \begin{cases} \max_{\tilde{c}_j^i, \tilde{c}_j^g, \tilde{h}_j^r} \log(\tilde{x}_j) - \psi^s \frac{n_j^{1+\zeta^s}}{1+\zeta^s} - F^s & \text{if } f = s \\ \max_{\tilde{c}_j^i, \tilde{c}_j^g, \tilde{h}_j^r} \log(\tilde{x}_j) - \psi^{m,1} \frac{(n_j^1)^{1+\zeta^{m,1}}}{1+\zeta^{m,1}} - \psi^{m,2} \frac{(n_j^2)^{1+\zeta^{m,2}}}{1+\zeta^{m,2}} - F^m & \text{if } f = m \end{cases} \quad (\text{B.10})$$

$$\tilde{x}_j \equiv \left(\sigma \tilde{c}_j^\eta + (1 - \sigma)(\tilde{h}_j^r)^\eta \right)^{1/\eta} \quad (\text{B.11})$$

As a subset of the full set of constraints for this optimization problem, the household considering being a renter in period t faces:

$$\tilde{c}_j^M + \tilde{c}_j^g + p_t^r \tilde{h}_j^r + \tilde{y}_{j+1} \Upsilon_A \leq (1 + r_t^p) \tilde{y}_j + b \tilde{e} q_t + \tilde{i}_{t,j}^{z,f} - \tilde{T}_{t,j}^{z,f} - \phi^o \tilde{h}_{j+1}^o \Upsilon_A - \tilde{\kappa}_j^{z,f} \quad (\text{B.12})$$

$$\tilde{h}_j^r \geq \underline{h}^r \quad (\text{B.13})$$

$$\tilde{y}_j \geq \underline{y}^{z,f} \quad (\text{B.14})$$

$$\tilde{h}_j^o = 0 \quad (\text{B.15})$$

Regardless of residential status, all households face the following equality and inequality constraints, employment fixed and variable costs, and initial and terminal conditions:

$$\tilde{y}_j \equiv \tilde{h}_j^o + \tilde{a}_j \quad (\text{B.16})$$

$$\tilde{c}_j \equiv (\tilde{c}_j^i)^{\theta^{z,f}} (\tilde{c}_j^g)^{(1-\theta^{z,f})} \quad (\text{B.17})$$

$$n_j^\epsilon = \begin{cases} 1 - l_j^\epsilon - n_j^h(n_j^\epsilon) & \forall j \leq R \\ 0 & \forall j > R \end{cases} \quad \epsilon = 1, 2 \quad (\text{B.18})$$

$$\tilde{i}_{t,j}^{z,f} \equiv \begin{cases} n_j \tilde{w}_t \mathbf{z}_j^{z,s} + \tilde{s} \tilde{s}_j^{z,s} & \text{if } f = s \\ (n_j^1 + \mu^z n_j^2) \tilde{w}_t \mathbf{z}_j^{z,m} + \tilde{s} \tilde{s}_j^{z,m} & \text{if } f = m \end{cases} \quad (\text{B.19})$$

$$\tilde{\kappa}_j^{z,f} = \begin{cases} \tilde{c} \tilde{c}^{z,s} \nu_j^{z,s} n_j & \text{if } f = s \\ \tilde{c} \tilde{c}^{z,m} \nu_j^{z,m} n_j^2 & \text{if } f = m \end{cases} \quad (\text{B.20})$$

$$F^s = \begin{cases} \phi^s & n_j > 0 \\ 0 & n_j = 0 \end{cases} \quad (\text{B.21})$$

$$F^m = \begin{cases} \phi^m & n_j^2 > 0 \\ 0 & n_j^2 = 0 \end{cases} \quad (\text{B.22})$$

$$\tilde{c}_j^i \equiv \begin{cases} \tilde{c}_j^M + \tilde{c}_j^h(n_j^h) & \text{if } f = s \\ \tilde{c}_j^M + \tilde{c}_j^{h,2}(n_j^{h,1}) + \tilde{c}_j^{h,1}(n_j^{h,2}) & \text{if } f = m \end{cases} \quad (\text{B.23})$$

$$\tilde{\mathcal{T}}_{t,j}^{z,f} \equiv t \tilde{a} x_{t,j}^{z,f} + \tau_t^a r_t^p \hat{\tilde{a}}_{t,j}^{z,f} + \tau_t^{pr} \hat{\tilde{i}}_{t,j}^{z,f} - (t \tilde{r} w_{t,j}^{z,f} + t \tilde{r} l_t - l \tilde{s} t_t) + (\tau_t^{sl} \hat{\tilde{i}}_{t,j}^{z,f} + \tau_t^{slp} \tilde{h}_{t,j}^o) \quad (\text{B.24})$$

$$\tilde{c}_j^i \geq \underline{c}^i \quad (\text{B.25})$$

$$\tilde{c}_j = \tilde{c}_j^i \quad \text{if } \tilde{c}_j^i = \underline{c}^i \quad (\text{B.26})$$

$$\tilde{y}_1 = \tilde{a}_1 \quad (\text{B.27})$$

$$\tilde{h}_1^o = \tilde{y}_{J+1} = 0 \quad (\text{B.28})$$

$$\mathbf{V}_{t,J+1}^{z,f} = 0 \quad (\text{B.29})$$

B.3 Equilibrium

For each household age cohort, j , productivity type, z , and family composition f , households have ordinary consumption, \tilde{c}^i , charitable giving, \tilde{c}^g , market labor hours, n , n^1 , and n^2 , owner-occupied housing services consumption, \tilde{h}^o , rental housing services consumption \tilde{h}^r , and future net worth \tilde{y}' , as control variables. Households have current net worth \tilde{y} as their endogenous individual state variable, and their age, productivity type, as family composition as their exogenous state variables. Endogenous aggregate state variables are effective market labor supply \tilde{N} , owner-occupied housing capital \tilde{H}^o , rental housing capital \tilde{H}^r , deposits \tilde{D} , private business capital \tilde{K} , public capital \tilde{G} , federal government debt \tilde{B} , and federal, state, and local tax instruments and transfer payments associated with given tax system, the set of which are denoted by \mathbb{T} . Then:

Definition 1. A *perfect-foresight stationary recursive equilibrium* is comprised of a measure of households $\tilde{\Omega}_{t,j}^{z,f}$, a value function $V_{t,j}^{z,f}(\tilde{y})$, a collection of household decision rules $\{\tilde{c}_{t,j}^{i;z,f}(\tilde{y}), \tilde{c}_{t,j}^{g;z,f}(\tilde{y}), n_{t,j}^{z,s}(\tilde{y}), n_{t,j}^{z,m,1}(\tilde{y}), n_{t,j}^{z,m,2}(\tilde{y}), \tilde{h}_{t,j}^{o;z,f}(\tilde{y}), \tilde{h}_{t,j}^{r;z,f}(\tilde{y}), \tilde{y}_{t+1,j+1}^{z,f}(\tilde{y}); \tilde{a}_{t,j}^{z,f}(\tilde{y}), \tilde{c}_{t,j}^{h;z,f}(\tilde{y})\}$, prices $\{\tilde{w}_t, r_t, p_t^r, \rho_t, r_t^p\}$, aggregates $\{\tilde{N}_t, \tilde{H}_t^o, \tilde{H}_t^r, \tilde{D}_t, \tilde{K}_t, \tilde{G}_t, \tilde{B}_t, \tilde{C}_t, \tilde{I}_t, \tilde{G}_t\}$, and the set of tax instruments and transfers \mathbb{T} associated with given tax system such that:

1. Household decision rules are the solutions to the constrained optimization problem in equations (B.1)-(B.29).
2. Macroeconomic aggregates are consistent with household behavior such that:

$$\begin{aligned}\tilde{N}_t &= \int_{\mathbb{Z}} \int_{\mathbb{J}} \tilde{\Omega}_{t,j}^{z,s} z_j^{z,s} n_{t,j}^{z,s}(\tilde{y}) + \tilde{\Omega}_{t,j}^{z,m} z_j^{z,m} (n_{t,j}^{z,1}(\tilde{y}) + n_{t,j}^{z,2}(\tilde{y})) \, dj \, dz \\ \tilde{H}_t^o &= \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \tilde{h}_{t,j}^{o;z,f}(\tilde{y}) \, dj \, dz \\ \tilde{H}_t^r &= \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \tilde{h}_{t,j}^{r;z,f}(\tilde{y}) \, dj \, dz \\ \tilde{D}_t &= \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \tilde{a}_{t,j}^{z,f}(\tilde{y}) \, dj \, dz \\ \tilde{C}_t &= \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \left((\tilde{c}_{t,j}^{i;z,f}(\tilde{y}) - \tilde{c}_{t,j}^{h;z,f}(\tilde{y})) + \tilde{c}_{t,j}^{g;z,f}(\tilde{y}) + \kappa_{t,j}^{z,f} \right) \, dj \, dz + \tilde{c}_t^{eol}\end{aligned}$$

3. Firms hire private factors of production to produce output at profit-maximizing levels while taking public capital and factor prices consistent with the following as given:

$$\tilde{w}_t = (1 - \alpha - g) \tilde{G}_t^g \tilde{K}_t^\alpha \tilde{N}_t^{-\alpha-g}$$

$$r_t = \tilde{K}_t^{-1} \left((1 - \tau_t^{bus})(1 - \tau_t^{slb})(\alpha + g)(\tilde{G}_t^g \tilde{K}_t^\alpha (A_t N_t)^{1-\alpha-g}) + \tau_t^{bus} l \tilde{s} d_t^{bus} \right)$$

4. The asset market clears such that:

$$\tilde{D}_{t+1} = \tilde{K}_{t+1} + \tilde{H}_t^r (\Upsilon_P \Upsilon_A)^{-1} + \tilde{B}_{t+1}$$

where the financial intermediary optimally allocates deposits into productive private capital and rental housing so that the following no-arbitrage condition holds:

$$p_t^r = r_{t+1} - \delta^K + \delta^r - \left(\frac{\partial \tilde{\Xi}_t}{\partial \tilde{K}_{t+1}} + \frac{\partial \tilde{\Xi}_{t+1}}{\partial \tilde{K}_{t+1}} \right)$$

and is willing to accept ‘safe-asset’ pricing of federal government bonds so that:

$$\rho_t = r_t - \varpi$$

Furthermore, the rate of return paid to households on deposits is determined by application of a zero profit condition so that:

$$r_t^p = \tilde{D}_t^{-1} \left((r_t - \delta^K) \tilde{K}_t - \tilde{\Xi}_t + p_t \tilde{H}_t^r - \delta^r \tilde{H}_{t-1}^r (\Upsilon_P \Upsilon_A)^{-1} + \rho_t \tilde{B}_t \right)$$

5. The goods market clears such that:

$$F(\tilde{G}, \tilde{K}_t, \tilde{N}_t) = \tilde{C}_t + \tilde{I}_t + \tilde{\mathcal{G}}_t$$

where private aggregate investment is defined as:

$$\tilde{I}_t \equiv \tilde{I}_t^K + \tilde{I}_t^o + \tilde{I}_t^r + \tilde{\Phi}_t^H$$

with:

$$\begin{aligned} \tilde{I}_t^K &= \tilde{K}_{t+1}(\Upsilon_P \Upsilon_A) - (1 - \delta^K) \tilde{K}_t + \Xi_t \\ \tilde{I}_t^o &= \tilde{H}_{t+1}^o(\Upsilon_P \Upsilon_A) - (1 - \delta^o) \tilde{H}_t^o \\ \tilde{I}_t^r &= \tilde{H}_t^r - (1 - \delta^r) \tilde{H}_{t-1}^r (\Upsilon_P \Upsilon_A)^{-1} \\ \tilde{\Phi}_t^H &= \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \left(\phi^o \tilde{h}_{t+1,j+1}^{o;z,f}(\tilde{y}) + \phi^r \tilde{h}_{t+1,j+1}^{r;z,f}(\tilde{y}) \right) dj dz \end{aligned}$$

and where aggregate government expenditures is defined as:

$$\tilde{\mathcal{G}}_t \equiv \tilde{C}_t^{fed} + \tilde{C}_t^{sl} + \tilde{I}_t^{fed} + \tilde{I}_t^{sl}$$

with:

$$\begin{aligned} \tilde{I}_t^{fed} &= \tilde{G}_{t+1}^{fed}(\Upsilon_P \Upsilon_A) - (1 - \delta^g) \tilde{G}_t^{sl} \\ \tilde{I}_t^{sl} &= \tilde{G}_{t+1}^{sl}(\Upsilon_P \Upsilon_A) - (1 - \delta^g) \tilde{G}_t^{sl} \end{aligned}$$

6. The federal government’s debt follows the law of motion:

$$\tilde{B}_{t+1}(\Upsilon_P \Upsilon_A) = \tilde{C}_t^{fed} + \tilde{I}_t^{fed} - (\tilde{T}_t^{hh} + \tilde{T}_t^{bus} + \tilde{T}_t^{beq}) + (1 + \rho_t) \tilde{B}_t$$

and maintains a fiscally sustainable path so that:

$$\lim_{k \rightarrow \infty} \frac{\tilde{B}_{t+k}}{\prod_{s=0}^{k-1} (1 + \rho_{t+s})} = 0$$

where net federal tax receipts from households, firms, and bequests are:

$$\tilde{T}_t^{hh} = \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \left(\tilde{tax}_{t,j}^{z,f} + \tau_t^a r_t^p \hat{a}_{t,j}^{z,f} + \tau_t^{pr} \hat{i}_{t,j}^{z,f} - (\tilde{tr}w_{t,j}^{z,f} + \tilde{tr}l_t - \tilde{l}st_t) - \tilde{s}s_{t,j}^{z,f} \right) dj dz$$

$$\tilde{T}_t^{bus} = \tau_t^{bus} \left((1 - \tau_t^{slb})(\tilde{Y}_t - \tilde{w}_t \tilde{N}_t) - \tilde{l}sd_t^{bus} \right)$$

$$\tilde{T}_t^{beq} = \tau_t^{beq} (1 - \Lambda) (\Upsilon_A) \int_{\mathbb{Z}} \int_{\mathbb{J}} (1 - \pi_j) \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \tilde{y}_{t+1,j+1} dj dz$$

7. The state and local composite government maintains a balanced budget:

$$\tilde{T}_t^{slh} + \tilde{T}_t^{slb} = \tilde{C}_t^{sl} + \tilde{I}_t^{sl}$$

where net state and local tax receipts from households and firms are:

$$\tilde{T}_t^{slh} = \int_{\mathbb{Z}} \int_{\mathbb{J}} \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} (\tau_t^{sl} \hat{i}_{t,j}^{z,f} + \tau_t^{slp} \tilde{h}_{t,j}^o) dj dz$$

$$\tilde{T}_t^{slb} = \tau_t^{slb} (\tilde{Y}_t - \tilde{w}_t \tilde{N}_t)$$

8. The measure of households is time-invariant:

$$\tilde{\Omega}_{t+1,j}^{z,f} = \tilde{\Omega}_{t,j}^{z,f}$$

9. The net worth of households that die before reaching the maximum age J is allocated to end-of-life consumption expenditures to bequests among the living such that:

$$\tilde{c}_t^{eol} = \Lambda (\Upsilon_A) \int_{\mathbb{Z}} \int_{\mathbb{J}} (1 - \pi_j) \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \tilde{y}_{t+1,j+1} dj dz$$

$$\tilde{beq}_t = (1 - \tau_t^{beq})(1 - \Lambda) (\Upsilon_A) \int_{\mathbb{Z}} \int_{\mathbb{J}} (1 - \pi_j) \sum_{f=s,m} \tilde{\Omega}_{t,j}^{z,f} \tilde{y}_{t+1,j+1} dj dz$$

Definition 2. A *steady-state perfect-foresight stationary recursive equilibrium* is a perfect-foresight stationary recursive equilibrium, where every growth-adjusted aggregate variable is time invariant.

C Description of Solution Algorithm

C.1 Steady State

1. Given the set of tax instruments and transfers \mathbb{T} and a starting guess for accidental bequests \tilde{beq}_t , make a guess for the set of endogenous aggregate state variables $\{\tilde{N}_t, \tilde{H}_t^o, \tilde{H}_t^r, \tilde{D}_t, \tilde{K}_t, \tilde{G}_t, \tilde{B}_t\}$, and use these guesses to compute the set of prices $\{\tilde{w}_t, r_t, p_t^r, \rho_t, r_t^p\}$.
2. For each household family composition $f = s, m$ and labor productivity type $z \in \mathbb{Z}$, obtain the optimal household decision rules associated with the value function in equation (B.1) at each age $j \in \mathbb{J}$ by backwards recursive iterations, subject to equations (B.2)-(B.29):
 - (a) Beginning with the maximum age J , analytically compute the optimal choices of housing, ordinary consumption, and charitable giving in terms of a grid of discrete values for current net worth \tilde{y}_J , both for the optimization problem particular to current period renters and homeowners as described in Section B.2.¹⁵ Use these choices to compute the value functions associated with renters and homeowners respectively, setting the value function to a large negative number for those choices that violate inequality constraints. For each net worth node, set $\mathbf{V}_{t,J}^{z,f}(\tilde{y}_J)$ equal to the maximum of $\mathbf{V}_{t,J}^{o;z,f}$ and $\mathbf{V}_{t,J}^{r;z,f}$. Store the household choices associated with the optimal housing status at each net worth node.
 - (b) For ages $j \in \{J-1, \dots, 1\}$, repeat part (a) for every possible $(\tilde{y}_j, \tilde{y}_{j+1})$ combination of net worth nodes to obtain the optimal household choices each age.
3. Use the optimal household decision rules obtained from the previous step to simulate lifecycle choices for each demographic with the initial conditions for net worth of $\tilde{y}_1 = \tilde{a}_1$ and owner-occupied housing $\tilde{h}_1^o = 0$. Simulation includes ne endowment levels from each demographic for a total of $nf \times nz \times J \times ne$ simulations. Compute all aggregates and implied prices.
4. Compare the new set of aggregates $\{\tilde{N}_t, \tilde{H}_t^o, \tilde{H}_t^r, \tilde{D}_t, \tilde{K}_t, \tilde{G}_t, \tilde{B}_t\}$ and accidental bequests \tilde{beq}_t to the initial guesses. If the value of these new aggregates are sufficiently close to the guessed values, then a steady-state equilibrium is obtained and the program can be terminated. If not, update each guess by taking a linear combination of the original guess and the new value obtained from application of equilibrium conditions. Use these new guesses to compute new values for the set of prices $\{\tilde{w}_t, r_t, p_t^r, \rho_t, r_t^p\}$. Return to step 2.

¹⁵Since the model is solved backwards, optimal choices for a household aged $j+1$ are known before those of a household aged j . Therefore $\tilde{h}_{t,j+1}^o$ and $\tilde{c}_{t,j+1}^g$ are used as proxies for $\tilde{h}_{t,j}^o$ and $\tilde{c}_{t,j}^g$ for purposes of computing itemized tax deductions during the optimization step. This allows for a substantial reduction in computational burden by avoiding a search over some specified support for \tilde{h}^o and \tilde{c}^g in addition to the search needed over labor supply in order to know $i_{t,j}^{z,f}$ for the working-age population. The approximation error introduced by this proxy is limited both the share of households which itemize tax deductions, and the lifecycle smooth path of these variables.

C.2 Transition Path

1. Choose the number of transition periods, tp , sufficiently large so that the economy reaches a steady state following a policy change.
2. Compute the initial steady state for the policy baseline and the final steady state under the new policy.
3. Provide an initial guess for the time path of the set of aggregate variables.
4. Compute the transition path.
 - (a) For years before policy change, use decision rules from steady state.
 - (b) Starting the first year of policy change, compute new decision rules for agents of each demographic for the price guesses for that time period.
 - (c) Run simulation starting J years before policy change so that by the first transition year, there are J ages. Use new decision rules and price guesses to simulate forward tp periods.
 - (d) Compute new aggregates and prices for each time period.
5. Compare the initial guess for the time path of the aggregate variables to their new time path obtained from the previous step. If time paths are sufficiently close, terminate the program. If not, update the guesses by taking a linear combination of the new time path and the old time path. Return to Step 4.

D Tables

Table A1: Targeted and Baseline Actual Employment Status by Type of Worker

Type of Worker	Data			Model: ITC			Model: BTF		
	FT	PT	U	FT	PT	U	FT	PT	U
Single	0.61	0.24	0.15	0.61	0.24	0.15	0.62	0.24	0.15
Married Primary	0.90	0.08	0.25	0.90	0.10	0.00	0.91	0.09	0.00
Married Secondary	0.42	0.32	0.26	0.42	0.32	0.26	0.43	0.33	0.25

Totals may not sum to 1 due to rounding

Table A2: Targeted and Baseline Actual Aggregate Ratios

Target Ratio	Data	Model: ITC	Model: BTF
Homeownership ratio	0.64	0.65	0.65
Private business investment to total private investment ratio	0.47	0.47	0.47
Private business investment to output ratio	0.16	0.17	0.17

Table A3: Select Exogenous Parameters

Demographics		
Terminal ages	R, J	40, 66
Rate of population growth	v_P	0.0075
Production		
Rate of technological progress	v_A	0.0118
Private capital share of output	α	0.3265
Public capital share of output	g	0.0352
Private capital depreciation rate	δ^K	0.0799
Private capital adjustment cost parameter	ξ^K	6
Housing		
Owner-occupied housing minimum down-payment	γ	0.20
Housing status adjustment cost	ϕ^o, ϕ^r	0.05, 0.05
Housing services depreciation rate	δ^o, δ^r	0.0555, 0.0570
Owner-occupied housing minimum (ITC)	\underline{h}^o	1.22
Owner-occupied housing minimum(BTF)	\underline{h}^o	1.14
Preferences		
Subjective discount factor	β	0.940
Non-housing consumption share of composite	σ	0.187
Housing/non-housing consumption substitution elasticity	η	0.487
Utility curvature parameter	$\zeta^{f,\epsilon}$	5
Intensive labor margin disutility (ITC)	$\psi^s, \psi^{m,1}, \psi^{m,2}$	395.1, 264.6, 176.7
Intensive labor margin disutility (BTF)	$\psi^s, \psi^{m,1}, \psi^{m,2}$	396.3, 279.9, 177.6
Extensive labor margin fixed cost (ITC)	ϕ^s, ϕ^m	0.354, 0.155
Extensive labor margin fixed cost (BTF)	ϕ^s, ϕ^m	0.393, 0.151
Government		
Public capital depreciation rate	δ^g	0.0317

Table A4: Trend Stationary Transformations for Selected Variables**Aggregate Variables:**

$$\begin{aligned}\tilde{C}_t^i &\equiv \frac{C_t^i}{A_t \mathbf{P}_t} & \tilde{H}_t^o &\equiv \frac{H_t^o}{A_t \mathbf{P}_t} & \tilde{H}_t^r &\equiv \frac{H_t^r}{A_t \mathbf{P}_t} & \tilde{D}_t &\equiv \frac{D_t}{A_t \mathbf{P}_t} & \tilde{K}_t &\equiv \frac{K_t}{A_t \mathbf{P}_t} \\ \tilde{C}_t^g &\equiv \frac{C_t^g}{A_t \mathbf{P}_t} & \tilde{B}_t &\equiv \frac{B_t}{A_t \mathbf{P}_t} & \tilde{G}_t &\equiv \frac{G_t}{A_t \mathbf{P}_t} & \tilde{T}_t &\equiv \frac{T_t}{A_t \mathbf{P}_t} & \tilde{N}_t &\equiv \frac{N_t}{\mathbf{P}_t}\end{aligned}$$

Individual Variables:

$$\begin{aligned}\tilde{c}_t^i &\equiv \frac{c_t}{A_t} & \tilde{h}_t^o &\equiv \frac{h_t^o}{A_t} & \tilde{h}_t^r &\equiv \frac{h_t^r}{A_t} & \tilde{c}_t^g &\equiv \frac{c_t^g}{A_t} & \tilde{a}_t &\equiv \frac{a_t}{A_t} \\ \tilde{y}_t &\equiv \frac{y_t}{A_t} & \tilde{i}_t &\equiv \frac{i_t}{A_t} & \tilde{\mathcal{T}}_t &\equiv \frac{\mathcal{T}_t}{A_t} & t\tilde{r}s_t &\equiv \frac{tr s_t}{A_t} & \tilde{s}s_t &\equiv \frac{ss_t}{A_t} \\ t\tilde{a}x_t &\equiv \frac{tax_t}{A_t} & b\tilde{e}q_t &\equiv \frac{beq_t}{A_t} & \tilde{w}_t &\equiv \frac{w_t}{A_t} & \tilde{\Omega}_t &\equiv \frac{\Omega_t}{\mathbf{P}_t}\end{aligned}$$